Agricultural productivity and soil carbon dynamics: a bioeconomic model*

Julia Berazneva[1,†], Jon Conrad[1], David Güereña[2], and Johannes Lehmann[2]

[1] *Charles H. Dyson School of Applied Economics and Management, Cornell University*
[2] *Crop and Soil Sciences, Cornell University*

31 October 2014

Abstract

We develop a dynamic bioeconomic model of agricultural households to investigate the likely effects of changes in agricultural practices on the natural resource base and on farmer livelihoods. Our theoretical modeling framework extends the traditional agricultural household model to incorporate the dynamic nature of natural resource management and to integrate biophysical processes through soil carbon management. Using an eight-year panel data set from an agronomic "chronosequence" experiment and data from household and market surveys in the western Kenyan highlands, our empirical model combines an econometrically estimated production function and a calibrated soil carbon flow equation in a maximum principle framework. We use the model to determine the optimal management of the farming system over time in terms of the application rates of mineral fertilizer and crop residues, taking into consideration initial resource endowments and prices. The optimal management strategies lead to soil carbon stocks of 20.2-40.2 Mg/ha (in the top 0.1 m) and maize yields of 3.5-4.2 Mg/ha, with discount rates of five to fifteen percent. The optimal application rates of mineral fertilizer and organic resources are, however, considerable — far higher than the current practices of western Kenyan farmers. They also depend on the initial condition of soil fertility, with more depleted soils requiring higher application rates at the outset. The equilibrium value of soil carbon is high and ranges between 108 and 148 US$/Mg, depending on the discount rate used, which highlights the considerable local private benefits of soil carbon sequestration. Annual soil carbon sequestration rates of 630 and 117 kg/ha can be achieved on depleted and medium-fertility soils, respectively.

Keywords: natural resource management, environmental degradation, agricultural productivity, bioeconomic model, soil carbon dynamics, Kenya. **JEL codes:** O13, Q12, Q24, Q56.

---

*We are grateful to David Lee for many and varied contributions to this research and this paper; to Dominic Woolf for developing the procedure used to calibrate the Rothamsted Carbon Model and estimate the equilibrium levels of soil carbon; and to survey enumerators and support staff at ICRAF-Kisumu. For valuable input and comments on an earlier draft, we thank Chris Barrett, Leah Bevis, Teevrat Garg, George Jakubson, Richard Klotz, Frank Place, Megan Sheahan, Thea Whitman, and participants at seminars at Cornell University and the 2014 Agricultural and Applied Economics Association annual meeting. The research described in this article was supported by the World Agroforestry Center (ICRAF) and the David R. Atkinson Center for a Sustainable Future, Cornell University. We also gratefully acknowledge financial support by the *Fondation des Fondateurs*, the U.S. National Science Foundation's Coupled Natural and Human Systems Program of the Biocomplexity Initiative (under grant BCS-0215890) and Basic Research for Enabling Agricultural Development Program (under grant IOS-0965336), and the Rockefeller Foundation (under grant No. 2004 FS 104). Additional financial support was provided by an AAEA Tweeten Scholarship, the Mario Einaudi Center for International Studies, Cornell University, and the Presbyterian Fund of Ithaca. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the funding agencies.

†Correspondence to: Julia Berazneva, 438 Warren Hall, Charles H. Dyson School of Applied Economics and Management, Cornell University, Ithaca, NY 14853, USA. E-mail: jb793@cornell.edu. Phone: +1-617-584-7629.

1 Natural resources and poverty

The world's poorest billion people mostly live in rural areas, where their livelihoods depend directly on natural capital (Hassan, Scholes, and Ash 2005). They derive a substantial portion of their livelihoods from farmland, forests, and waterways, so that the natural capital is not only an amenity but also a primary factor of production. While natural resources are typically renewable, overuse or mismanagement leads to their depletion and deterioration (Dasgupta 2010). Crop lands stop responding to applications of fertilizer, inland and coastal fisheries become depleted, and excessive deforestation leads to loss of biodiversity. For the poor, especially in fragile environments in Africa and some parts of Latin America and Asia, the reduced stocks and the deteriorated ecosystem services they provide lead to lower land productivity and stagnating incomes, as well as discourage investments in maintaining the natural resource base or in new agricultural technologies needed for sufficient food production. Improving the management of natural resources is, therefore, fundamental to addressing rural poverty.

Management of soil resources in smallholder agriculture of Sub-Saharan Africa (SSA) is of particular concern as depletion of soil fertility is considered to be one of the major biophysical causes of low per capita food production (Sanchez 2002). About 414 million people in the region – up from 290 million people in 1990 – live in extreme poverty, and Sub-Saharan Africa remains the region with the highest prevalence of undernourishment (UN 2014). The neglect of the rural sector by governments and the collapse of traditional societies and their practices (e.g., fallowing) over many decades have resulted in the removal of large quantities of nutrients from soils without sufficient quantities of fertilizer or organic resources to replenish them. As soils depleted of organic matter are less responsive to mineral fertilizers and changing climate, more and more resources are needed to maintain food production. As a result, smallholder farmers in SSA "cultivate marginal soils with marginal inputs, produce marginal yields, and perpetuate marginal living and poverty" (Lal 2004, p. 1626). In addition, degradation of soil resources in SSA leads to numerous environmental concerns, such as water pollution and conversion of natural environments and marginal lands to agriculture, among many others.

Overcoming soil fertility depletion across the continent requires the combined application of mineral fertilizer and organic resources (Vanlauwe and Giller 2006). The two inputs fulfill different

functions. While the main role of mineral inputs is to supply nutrients, organic resources replenish soil carbon and soil organic matter stocks that enhance soil physical, chemical, and biological processes and properties, fundamental for long-term soil fertility and nutrient use efficiency (Blanco-Canqui et al. 2013). The improvement of soil properties not only sustains yields, but also enhances the inherent capacity of soils to buffer against extreme climatic events such as droughts, heat waves, and floods. Moreover, the limited availability and high cost of external inputs, as well as competing uses for on-farm organic resources (fuel and fodder) often discourage the applications of either in sufficient quantities (Vanlauwe and Giller 2006). Understanding how soil resources respond to the combined applications and determining the optimal application rates is, therefore, important for improved resource allocation at the farm level and for national agricultural policy decisions.

To address both agricultural production and environmental concerns and to investigate the likely effects of changes in agricultural practices on the natural resource base and on farmer livelihoods, we develop a dynamic bioeconomic model of agricultural households in the western Kenyan highlands. Our theoretical bioeconomic modeling framework extends the traditional economic agricultural household model to incorporate the dynamic nature of natural resource management and to integrate biophysical processes through soil carbon management. Using an eight-year panel data set from an agronomic "chronosequence" experiment in Vihiga and Nandi districts in Kenya and data from household and market surveys, our model combines an econometrically estimated production function and a calibrated soil carbon flow equation in a maximum principle framework. We use the model to determine the optimal management of the farming system over time in terms of the application rates of mineral fertilizer and crop residues, taking into consideration initial resource endowments and prices. Moreover, the unique nature of our data set – a "false" time series, comparable to a quasi-natural experiment and not often available for economic research – allows for estimation of site-specific dynamic relationships between agronomic productivity, land use management decisions, and soil carbon.

We find that the optimal management strategies lead to carbon stocks of 20.2-40.2 Mg/ha (in the top 0.1 m) and maize yields of 3.5-4.2 Mg/ha, with discount rates of five to fifteen percent and given the prevailing price levels. Reaching and sustaining these carbon stocks and maize yields, however, requires different applications rates of mineral fertilizer and crop residues for farms with different initial resource endowments, with more depleted soils requiring higher application rates

at the outset. Our focus on soil carbon allows us to assess its value and, moreover, permits us to estimate the potential for and impacts of carbon sequestration in the research area. The equilibrium value of soil carbon is high and ranges between 108 and 148 US$/Mg of carbon or 29 and 40 US$/Mg of carbon dioxide equivalent, depending on the discount rate used, highlighting the considerable local private benefits of soil carbon sequestration. Annual soil carbon sequestration rates of 630 and 117 kg/ha can be achieved on depleted and medium-fertility soils, respectively.

The remainder of this paper is organized as follows. Section 2 provides a brief review of the existing literature focusing on valuing natural resources and agricultural production in Sub-Saharan Africa, discusses the implications of the static and dynamic analysis, and outlines our paper's contribution. Section 3 discusses the soil carbon resources and Section 4 introduces our research area, the western Kenyan highlands. Section 5 presents our theoretical model. The farmer's objective is to maximize the discounted present value of annual profits from a representative hectare planted with maize over an infinite horizon. Section 6 reports the construction of our empirical model, focusing on the econometric estimation of the underlying maize production function, calibration of the soil carbon equation, and the prices used in the analysis, together with introducing the data. Section 7 presents the results of the model's steady-state values, simulation for several policy scenarios of interest (current practices vs. optimal decision rules), and sensitivity analyses. Section 8 concludes the paper.

2 Value of soil resources in agricultural production

A number of qualitative studies focusing on the implications of rural environmental degradation have shed light on how the incomes of rural communities are affected by resource degradation. Yet, there is limited empirical work demonstrating the quantitative importance of natural resources as factors of production (Lopez 1997). Valuing the local natural resource base in the developing world is a challenging task. The subsistence nature of smallholder agriculture means that farmers make production, consumption, and resource allocation decisions subject to considerable temporal uncertainty, both in terms of the biophysical environment (soil quality, rainfall, adverse weather conditions, pests, etc.) and the economic environment (output and input prices, input availability, etc.). Hence, farming systems in developing countries are very complex; they include a large

number of interlinked activities, outcomes of which frequently depend directly on the conditions of local natural resources rather than on external input use. Since natural resources affect farming activities through complex biophysical processes, their effects may not be entirely understood and are difficult to quantify. In addition, natural resource management in developing countries has environmental implications that often go beyond farm boundaries, both temporally and geographically, and volatility in markets and prices complicate the analysis (Barbier and Bergeron 2001).

Interest in the impacts of soil resource management on agricultural output dates back to at least the Dust Bowl era of the 1930s in the American Midwest (Thampapillai and Anderson 1994). Recognizing the effects of land use practices and agricultural technologies on agricultural productivity, a rich body of literature in agricultural economics addresses these issues in a static production framework. Specifically for Sub-Saharan Africa, numerous studies address crop responses to fertilizer use, hybrid seeds, and other external inputs or reasons for their limited applications (see, for example, recent studies in Kenya by Marenya and Barrett (2009a), Suri (2011), Duflo, Kremer, and Robinson (2011), and Sheahan, Black, and Jayne (2013)).

At the same time, relatively few studies explicitly recognize the direct impacts of management practices on soil fertility beyond including indicators for farmers' perception of soil quality. Several studies consider the effects of decreasing common pools of biomass or leaving land fallow. Lopez (1997) examines the common village-level stock of biomass and its decline due to reductions in the fallow periods in Ghana. By incorporating village-level biomass as an input in agricultural production, he finds that it is an important factor of production and is often exploited beyond the socially optimal level. Goldstein and Udry (2008) demonstrate the importance of fallows for on-farm soil quality, and hence, profits from agricultural plots left fallow, also in Ghana. As applications of animal manure deliver not only nutrients necessary for plant growth but also help maintain long-term soil fertility, Gavian and Fafchamps (1996) and Sheahan, Black, and Jayne (2013), for example, include indicator variables for manure use, while Marenya and Barrett (2009b) consider the value of livestock as a proxy for manure applications. Only two papers from SSA, to our knowledge, include quantities of animal manure as inputs in crop response models (Teklewold 2012; Matsumoto and Yamano 2011), and only one considers the quantities of crop residues (Berazneva et al. 2014).

Soil degradation in the current period imposes a reduction in net benefits on the future generation (i.e., user cost); therefore, it ideally needs to be evaluated within an intertemporal framework.

Some of the earlier work treating soil as a renewable resource comes from the U.S. For example, Burt (1981) employs a dynamic programming framework to model soil conservation in the U.S. Northwest, using depth of top soil and the percentage of organic matter therein as the state variables and cropping practices as the control variables. McConnell (1983) uses soil loss as the state variable in a theoretical model of soil conservation to argue that the private and social rates of soil erosion are the same under most institutional arrangements.

Yet, the paucity of detailed information available to economists, such as technical data on rates of soil fertility loss, as well as the inherent complexity of farming systems, has limited the dynamic analysis of soils in developing countries. Some past studies focus on qualitative analysis (see, for example, French (1986)), while others include biophysical variables such as soil nutrients (carbon, nitrogen, phosphorus), organic matter depletion and soil erosion and simulate the effects of land degradation by incorporating parameters estimated in a separate biophysical model into a model of economic behavior. Barbier (1998), for example, combines a linear programming model of economic behavior with agricultural production parameters obtained from the results of the Erosion Productivity Impact Calculator (EPIC) model to study land degradation in Burkina Faso. Wise and Cacho (2011) use the Soil Changes Under Agriculture, Agroforestry, and Forestry (SCUAF) model to derive soil-carbon changes, crop-yield dynamics and tree-biomass accumulation as functions of management variables in a dynamic simulation model to assess the financial viability of agroforestry systems as carbon sinks in Indonesia.

Specifically for western Kenya, two simulation models examine the links between biophysical and economic processes at the farm scale. Shepherd and Soule (1998) develop a simulation model to predict the long-term effects of farming systems on nutrient cycling, plant production and farm income, while Stephens et al. (2012) use a system dynamics model to examine the interactions between natural resource-based poverty traps and food security for small farms in Kenya. Both of these models, however, are descriptive or predictive (not optimization problems), and do not allow for changes in management practices in response to physical changes in soil.

Only when detailed biophysical data from the research area are available and allow for estimation of crop productivity responses and soil condition changes, however, can bioeconomic models truly internalize the effects of land degradation on household intertemporal decisions. In one such example, using plot-level data from field experiments in the Ethiopian highlands, Holden, Shiferaw,

and Pender (2005) assess the impacts of land degradation, population growth, market imperfections, and increased risk of drought on household production, food security, and welfare.

Our paper contributes to both static and dynamic strands of the literature. Our dynamic bioeconomic model treats soil fertility as an input in agricultural production and a renewable resource, while the rich agronomic and socio-economic data sets we use allow for the model's precise estimation. We explicitly recognize that agricultural outcomes depend on the conditions of local natural resources and consider soil fertility as a factor of agricultural production. At the same time, we model farm household intertemporal management practices and their effects on current and future agricultural productivity. By doing so, we are also able to assign monetary value to soil fertility and specifically soil carbon, thus quantitatively demonstrating the importance of natural resources as primary factors of production in smallholder agriculture.

3 Focus on soil carbon

We focus on soil carbon as the interface between the social and biophysical processes. There are several compelling reasons for doing so. First, there is a strong relation between soil organic carbon (SOC) and soil fertility, on the one hand, and crop productivity and soil fertility, on the other. Although SOC is not essential to plant growth *per se* and its benefits can be to a degree substituted by applying external inputs, the SOC pool is related to the amount of soil organic matter (SOM) with multiple benefits to soil productivity, such as nutrient availability, water-holding capacity, and soil biota, and therefore, on agronomic/biomass productivity (Lal 2006; Blanco-Canqui et al. 2013). Increases in SOC, in addition, not only increase average yields, but also decrease the susceptibility of yields to weather shocks (Graff-Zivin and Lipper 2008).

Land use decisions, such as the conversion of forests to agricultural land, have a major influence on the level of the SOC pool. By removing the bulk of aboveground biomass, conversion to agricultural land breaks the carbon movement cycle in the ecosystem as soil carbon is not replenished fast enough to keep up with decomposition. A large fraction of the accumulated carbon and soil nutrients thus become lost; this generally happens most rapidly in the years immediately after agricultural conversion (Murty et al. 2002). Current agricultural practices in resource-poor economies also deplete the SOC and SOM pools, and by doing so degrade soil quality and have

an adverse effect on agronomic productivity. At the same time, agricultural management practices that alter the inputs of organic matter or the decomposition rate of SOM can build up the stock of soil organic carbon. Such practices include leaving land fallow, no-till farming, residue retention, manuring, composting, N fertilization, mulching, incorporation of grass and legumes in the rotation cycle, and the use of agroforestry systems (Lal 2006; WB 2012).

The second reason for our focus on soil carbon is found in the potential of carbon sequestration to simultaneously achieve two sustainability goals: the improvement of agricultural productivity and climate change mitigation (Antle and Stoorvogel 2008). Land use changes and agricultural practices can transfer atmospheric carbon dioxide (CO_2) to soil. With the continuing use of these practices over 20-50 years, observed rates of SOC sequestration can range from 0 to 150 kg C/ha per year in dry and warm regions, and up to 100-1,000 kg C/ha per year in humid and cool climates, depending on soil texture, characteristics and climate (Lal 2004). As a result, it is estimated that soil carbon sequestration from sustainable land use and agriculture could potentially offset the emissions from fossil fuels by 0.4-1.2 Gt C per year, or 5 to 15 percent of the global fossil fuel emissions (Lal 2004).

The third reason for focusing on soil carbon, as Antle and Stoorvogel (2008) note, is the fact that despite great interest in the international community and among national policy-makers, there is little available information about the potential for and impacts of payments for agricultural carbon sequestration from actual projects. By estimating the potential for carbon sequestration and valuing carbon on the western Kenyan farms, we provide such empirical evidence.

4 Study area: western Kenyan highlands

The western Kenyan highlands provide our case study. Surrounding Lake Victoria on the Kenyan side, this is one of the most densely populated and poorest regions of the country, with over 55 percent of the population living below the national rural poverty line (about US$0.59 per day) (WRI 2007). The complexity of smallholder farming systems in developing countries, as discussed above, also characterizes this research area. Rural households engage in a range of agricultural activities. While their main objective is increasing food supplies (Waithaka et al. 2006), subsistence farmers also strive to earn income and satisfy household energy needs.

Average farms are about 0.5-2 hectares in size and originally formed part of the Guineo-Congolese forest system that became converted to agricultural land in the twentieth century. Farms have medium to high agricultural potential (WRI 2007), but suffer from severe soil degradation. The incorporation of crop residues at plowing, crop rotations and short fallows were some of the main means of maintaining soil fertility in crop fields in western Kenya until the 1960s (Crowley and Carter 2000). As population increased and farm areas declined, however, crop rotations and fallowing periods were reduced and most farmers have stopped planting woodlots, making cereal residues the principal on-farm source of fuel. Only wealthier households with larger land holdings have continued fallowing and/or using crop rotations (Crowley and Carter 2000). As a result, the amount of organic material returned to the soil after harvest has significantly declined in the area and maize monoculture has hastened soil deterioration (Solomon et al. 2007).

We examine one of the main agricultural activities of households in rural western Kenya – production of a subsistence crop, maize (*Zea mays* L.). Maize is the most commonly grown and consumed grain in the area, having established itself as a dominant food crop in Kenya at the beginning of the 20th century (Crowley and Carter 2000). Despite its significant contribution to satisfying household food needs and cultivation by most smallholders on the largest proportion of farm area, maize production often results in low yields. Farmer-reported average maize yields are in the range of 1.65 and 2.7 Mg/ha[1] (Sheahan, Black, and Jayne 2013; Berazneva et al. 2014). The potential yields in response to water and nutrient availability for this environmental range, however, are found to be much higher, between 10.8 and 11.4 Mg/ha (Tittonell and Giller 2013).

Low yields correspond to current agricultural practices – no fallowing, limited use of hybrid seeds and mineral fertilizer, as well as organic resources, such as crop residues, animal manures, compost, etc. In fact, the use of maize residues as soil organic amendments is traded off against other applications. Less than half of all residues is left on the field for soil fertility management; the other half is roughly equally split between household energy and animal feed (Berazneva et al. 2014). Soil carbon stocks, as a result, are heavily depleted (Solomon et al. 2007).

Data from several sources are used to build the bioeconomic model described below. Plot-level maize yields and carbon stocks come from a long-term agronomic experiment in Vihiga and Nandi districts of western Kenya in 2005-2012, while socio-economic household-level data and prices are

[1] 1 megagram (Mg) = 1,000 kilogram (kg) = 1 metric tonne. 1 hectare (ha) = 10,000 square meters.

from the household and market surveys in the districts surrounding the agronomic sites in 2011-2013. The data sources are shown in Figure 1 and further discussed in the sections that follow. Our soil analysis reported the total stocks of soil carbon, which are equivalent to total organic carbon since the soils in the research site are acidic and do not contain carbonates.

5 Economic model

Our model is similar to that of Burt (1981): the farmer's objective is to maximize the discounted present value of net returns from the land resource over an infinite planning horizon. Instead of focusing on the depth of top soil and percentage of soil organic matter to capture soil fertility, however, we use soil carbon as a state variable that influences agronomic productivity, and its flow depends on the choice of farming practices. Adopting the model to a developing-country setting requires some additional considerations. When markets for agricultural outputs or inputs are missing, or when transaction costs are excessive, rural households respond by linking their production and consumption decisions to satisfy multiple objectives of food security, income generation and risk reduction (de Janvry, Fafchamps, and Sadoulet 1991). This nonseparability in consumption and production also implies that consumption needs and asset distribution have significant impacts on production decisions and thus the management of natural resources (Holden and Binswanger 1998). While we acknowledge that small farmers in our study area may not be able to maximize profits, similar to Wise and Cacho (2011), we argue that profit is an important part of the farmers' objectives. Moreover, we explicitly account for opportunity cost of household labor, land, and organic resources. In addition, we use farmer-reported prices that reflect potential transaction costs and availability constraints and perform price sensitivity analysis. We also only focus on production of maize, the staple cereal, and the management practices to manage soil fertility and increase maize yields, abstracting from the farmer's decisions in terms of the amount of land and the choice of crops to cultivate.

Assuming away the variation in farm size (and thus imposing constant returns to scale), suppose a farmer cultivates a hectare of land of homogenous quality with maize. Let c_t represent the state of farmer's land in year t, defined by a single soil-quality indicator – soil carbon content. The farmer grows maize by employing a range of land use and management decisions: let f_t be the quantity of

mineral nitrogen applied and $\alpha_t \in [0,1]$ be the share of maize residues left on the field for soil fertility at the end of year t that influences the stock of soil carbon in $t+1$. Let maize production (Mg/ha) be a function of soil carbon and nitrogen fertilizer: $y_t = y(c_t, f_t)$. The change in soil carbon content depends not only on the carbon content in the previous period, but also on the farmer's management decisions: $c_{t+1} - c_t = g(c_t, f_t, \alpha_t)$, where $g(\cdot)$ is a function describing soil carbon dynamics. The initial level of soil carbon, $c_0 = a > 0$, is given. Let $\pi_t = \pi(c_t, f_t, \alpha_t) = py(c_t, f_t) - nf_t - qr_t\alpha_t - m$ be the annual profit obtained from a representative hectare planted with maize, where p is the price of maize (\$/Mg), n is the price of nitrogen (\$/Mg), q is the per unit value of crop residues in highest household use (\$/Mg), r_t is the total quantity of maize residues produced in year t (Mg/ha), and m is the per-hectare cost of preparing the land, planting, and harvesting maize (\$/ha).

5.1 Farmer's objective

The farmer's objective is to maximize the discounted present value of annual profits by growing maize on a hectare of land over an infinite horizon, with a discount factor $\rho = 1/(1+\delta)$ for the discount rate δ:

$$\max_{\{f_t, \alpha_t\}} \pi = \sum_{t=0}^{\infty} \rho^t [py(c_t, f_t) - nf_t - qr_t\alpha_t - m]$$

$$\text{subject to}$$

$$c_{t+1} - c_t = g(c_t, f_t, \alpha_t),$$

$$c_0 = a > 0, \text{given.} \tag{1}$$

To simplify the exposition, it can be assumed that total crop residues produced in period t are a fraction of maize yield in period t, so that $r_t = ky_t$, where k is the time-independent conversion parameter (maize residues to grain ratio).[2] Then assuming that f_t, α_t, c_t and λ_t, the multiplier on the soil carbon constraint, are restricted to being nonnegative, the discrete-time current value

[2]Straw to grain ratio is a standard conversion parameter to estimate the production of crop residues (Smil 1999).

Hamiltonian can be written as

$$
\begin{aligned}
H &= py(c_t, f_t) - nf_t - qky(c_t, f_t)\alpha_t - m + \rho\lambda_{t+1}g(c_t, f_t, \alpha_t) \\
&= (p - qk\alpha_t)y(c_t, f_t) - nf_t - m + \rho\lambda_{t+1}g(c_t, f_t, \alpha_t), \qquad (2)
\end{aligned}
$$

where the multiplier λ_{t+1} can be interpreted as the current-value shadow price of the soil carbon stock at time $t+1$. The first order conditions require that

$$
\begin{aligned}
\frac{\partial H}{\partial f_t} &= (p - qk\alpha_t)\frac{\partial y(\cdot)}{\partial f_t} - n + \rho\lambda_{t+1}\frac{\partial g(\cdot)}{\partial f_t} = 0, && (3) \\
\frac{\partial H}{\partial \alpha_t} &= -qky(\cdot) + \rho\lambda_{t+1}\frac{\partial g(\cdot)}{\partial \alpha_t} = 0, && (4) \\
\rho\lambda_{t+1} - \lambda_t &= -\frac{\partial H}{\partial c_t} = -\left[(p - qk\alpha_t)\frac{\partial y(\cdot)}{\partial c_t} + \rho\lambda_{t+1}\frac{\partial g(\cdot)}{\partial c_t}\right], && (5) \\
c_{t+1} - c_t &= \frac{\partial H}{\partial(\rho\lambda_{t+1})} = g(c_t, f_t, \alpha_t). && (6)
\end{aligned}
$$

Re-writing the first order conditions, we have the following results:

$$
\begin{aligned}
(p - qk\alpha_t)\frac{\partial y(\cdot)}{\partial f_t} + \rho\lambda_{t+1}\frac{\partial g(\cdot)}{\partial f_t} &= n, && (7) \\
\rho\lambda_{t+1}\frac{\partial g(\cdot)}{\partial \alpha_t} &= qky(\cdot), && (8) \\
\lambda_t = \rho\lambda_{t+1}\left[1 + \frac{\partial g(\cdot)}{\partial c_t}\right] &+ (p - qk\alpha_t)\frac{\partial y(\cdot)}{\partial c_t}, && (9) \\
c_{t+1} - c_t &= g(c_t, f_t, \alpha_t). && (10)
\end{aligned}
$$

Equation 7 and 8 equate "full marginal value" to marginal cost for the two management variables, f and α. Full marginal value is the marginal value product in current production plus the marginal value based on the discounted shadow price for carbon in $t+1$. Equation 9 is a form of the co-state equation relating the shadow price on carbon in period t to its discounted future marginal value in $t+1$ plus the marginal value product of carbon in production in period t. Equation 10 is a restatement of the state equation.

6 Empirical model

The construction of the empirical model used to estimate the farmer's maximization problem (Equation 1) consists of several steps. We first specify and econometrically estimate the maize yield equation ($y(\cdot)$) as a function of soil carbon stock (to a depth of 0.1 m) (c) and nitrogen fertilizer (f). We then specify and calibrate the soil carbon equation ($g(\cdot)$) to approximate the annual change in soil carbon stock from maize residues left on the field and carbon loss from mineralization. Two management variables of interest are, then, the application rates of nitrogen fertilizer (f) and the share of maize residues left on the field as soil amendments to replenish soil carbon (α). The two equations interactively describe crop-yield dynamics and soil-carbon changes and provide parameters for our biophysical model. As a last step, we describe the economic variables used in the empirical model and their sources before proceeding to the discussion of our results.

6.1 Maize yield function

The biophysical data used to estimate the maize yield function come from agronomic experimental sites in Vihiga and Nandi districts of western Kenya. The sites were established in 2005 and maintained until 2012 as a part of a chronosequence experiment designed to analyze the long-term effects of land conversion from primary forest to continuous agriculture (Ngoze et al. 2008; Kinyangi 2008; Kimetu et al. 2008; Guerena 2014). A chronosequence – a set of sites that share similar attributes but are of different ages – was established on twenty-eight farms of different age since conversion from forest to agricultural land.[3] Prior to the establishment of the experimental sites, soils had received very little or no mineral fertilizer since forest clearing, no animal manure, and had been cropped with maize for 5, 20, 35, 80, and 105 years (Kinyangi 2008).

Each year experimental plots received nitrogen mineral fertilizer at a rate of 0 and 120 kg per hectare, and in 2011 and 2012 the nitrogen (N) application rate varied at 0, 80, 120, 160, 200, and 240 kg per hectare. In addition, organic inputs (*Tithonia diversifolia* leaves, wood charcoal, and sawdust) were applied at a rate of 18 Mg of carbon per hectare over three seasons in 2005 and 2006 (6 Mg/ha per season). All other management variables (e.g., type of maize hybrid seed, timing of weeding and harvesting, etc.) were maintained the same across the sites. The four treatments

[3] A chronosequence is an important tool for studying temporal dynamics of soil development across multiple time-scales (Stevens and Walker 1970).

include control (with and without N input), *Tithonia diversifolia* (with and without N input), charcoal (with and without N input), and sawdust (with and without N input). Maize grain yield data (oven-dry measurements) are available for each farm, treatment and year from 2005 to 2012. While the soil samples were collected at 0.1 m depth from experimental plots each year, not all were analyzed for soil carbon stock given the large number of samples involved and therefore the high time and financial cost of soil chemical analysis. A sub-sample, representing three major age groups, each treatment and year (177 samples), was analyzed for total soil carbon after ball milling using a Dumas combustion analyzer. Bulk density was measured using three rings per plot in order to calculate carbon stocks. Following Kinyangi (2008), the resulting data were fitted using a three-parameter exponential decay model, $y = y_0 + ae^{-bx}$, where y_0 is the final carbon equilibrium level, a is the amount of carbon lost (Mg/ha), b is the rate of loss, and x is years since conversion from forest, for each of treatment-fertilizer sub-samples. The established relationships were then used to predict plot-specific soil carbon stocks.[4]

The heterogeneity of soil fertility and the differing impacts of inputs on individual plants on otherwise homogenous farms have been shown to imply a smooth aggregate production function (Berck and Helfand 1990). We use a generalized quadratic specification to approximate the unknown true relationship between maize yields, soil carbon stocks and nitrogen applications, and to capture the interactions between soil fertility and nitrogen inputs. The same functional form is also used in recent studies focusing on maize production and soils across Sub-Saharan Africa (see, for example, Sheahan, Black, and Jayne (2013) and Harou et al. (2014)). We estimate

$$y_{kit} = \gamma_0 + \gamma_c c_{kit} + \gamma_{cc} c_{kit}^2 + \gamma_f f_{kit} + \gamma_{ff} f_{kit}^2 + \gamma_{cf} c_{kit} f_{kit} + \eta_k + \zeta_i + \theta_t + \xi_{kt} + \epsilon_{kit}, \tag{11}$$

where y_{kit} is maize yield (Mg/ha) for treatment k on farm i at time t, c_{kit} is the soil carbon stock (Mg/ha), f_{kit} is the nitrogen fertilizer input (Mg/ha), γ_0, γ_c, γ_{cc}, γ_f, γ_{ff} and γ_{cf} are coefficients to be estimated, η_k is a treatment fixed effect, ζ_i is a farm-level fixed effect, θ_t is a year fixed effect, ξ_{kt} is a treatment-year interaction fixed effect, and ϵ_{kit} is the i.i.d., mean zero, normally distributed regression error.

[4]Soil samples were collected at harvest of the long rains season, so that in the final predictions a lagged variable is used: e.g., a soil carbon stock measured during the harvest of 2005 is used as $c_{t=2006}$, a soil carbon stock relevant for maize production in 2006. The sampling procedure and the construction of soil carbon stock variable are described in Appendix A.1.

Pooling observations across the eight years, twenty-eight farms, and four treatments, there are 1,450 observations.[5] Table 1 shows summary statistics for the variables used.[6] Similarly to Harou et al. (2014), our data and, therefore, estimations focus on locally attainable yield levels, which are defined as the yields from researcher-managed plots or the maximum yields achievable by resource endowed farmers in their most productive fields (95th-percentile yields in a farmer field survey) (Tittonell and Giller 2013). The average maize yields from the chronosequence experiment are 4.5 Mg/ha, while they are 4.38 Mg/ha for the 95th-percentile of farmers in the household survey conducted in the same area. Our detailed data set spanning eight years allows us to make credible estimates of the yield response rates to the additions of mineral fertilizer and soil carbon stocks. It does not allow, however, for the incorporation of legume intercropping and additions of animal manure as soil amendments that are also characteristic of the western Kenyan highlands.

Table 2 shows the estimated coefficients of the generalized quadratic function, as well as the average marginal productivities for carbon stock and nitrogen fertilizer. The estimation includes farm, year and treatment fixed effects, which control for initial conditions and time-invariant farm heterogeneity (i.e., slope, drainage, etc.), annual changes in rainfall and other weather effects, and the treatments, respectively. Since rainfall may have differential impacts on plots with different treatments, we also include the year-treatment interaction. Column (1) shows the standard errors clustered at the farm level, while column (2) shows bootstrapped standard errors.

The quadratic specification seems to fit the data quite well; the R-squared of the estimated model is 0.50, and we cannot reject the joint significance of the second order terms (a Wald test statistic of 8.16 and a P-value of zero against the $\chi^2(3)$ distribution). For every additional Mg of soil carbon stock, maize yield increases by 0.06 Mg,[7] while an addition of 100 kg of nitrogen fertilizer results in the yield increase of 1,039 kg. Figure 2 shows the distribution of the estimated returns to soil carbon and nitrogen application, with the estimated returns to nitrogen being shown for observations with $f > 0$. Of particular interest is the negative coefficient on the interaction term between carbon

[5]The number of observations differs by farm. Several farms exited the chronosequence experiment because of their change in land ownership or farmers decided to discontinue working with the researchers. Some other observations are missing due to outlier status in grain yield measurements or other recording issues. We omit observations with the values of maize yield in the top and bottom one percent of the distribution.

[6]Nitrogen content of maize roots and residues is very low; it averages around 1 percent (for example, 0.7 percent in Gentile et al. (2011), 1.06 percent in Kinyangi (2008), or 1.27 percent in Latshaw and Miller (1924)). We do not add nitrogen in maize residues to nitrogen from mineral fertilizer.

[7]Diaz-Zorita, Duarte, and Grove (2002) find a similar relationship: a 1 Mg/ha decrease in SOC is associated with a 0.04 Mg/ha yield loss across 134 farmers' wheat fields in Argentina.

stock and nitrogen fertilizer. The negative sign suggests substitutability between the two inputs, which is similar to the findings of a comprehensive meta analysis of Chivenge, Vanlauwe, and Six (2011). Synthesizing the results from the 57 agronomic studies of maize yields on smallholder farms across Sub-Saharan Africa, the authors find that overall, the combined addition of organic resources and nitrogen fertilizer results in negative interactive effects on maize yields (about 445 kg/ha as compared to 218 kg/ha in our sample). They argue that the negative interaction effects can be explained by an excess amount of nitrogen added: over 70 percent of the studies included in their meta analysis applied at least 100 kg of nitrogen per hectare, which can reduce the agronomic N use efficiency and conceal the possible positive interactions.[8] The N application rate in our sample is 120 kg N/ha for most observations and we find similar agronomic N use efficiency: 12.3 kg of maize grain per 1 kg of nitrogen added, similar to the 13.6 kg of maize grain estimated by Chivenge, Vanlauwe, and Six (2011).[9] Our data and estimation also support the finding of Chivenge et al. (2007) and Chivenge, Vanlauwe, and Six (2011) that the combined applications of organic resources and nitrogen results in lower soil organic carbon than the addition of organic resources alone. This may be attributed to enhanced decomposition of the added organic resources (Knorr, Frey, and Curtis 2005; Khan et al. 2007; Kimetu et al. 2009).[10]

Based on the estimated coefficients in Table 2, the maize production function estimated for the bioeconomic model is the following:[11]

$$y_t = -0.810 + 0.113c_t - 0.000413c_t^2 + 27.04f_t - 41.30f_t^2 - 0.218c_t f_t. \tag{12}$$

[8]Marenya and Barrett (2009b), in contrast, find complementaries between soil carbon and nitrogen fertilizer in the same research area as ours. N applications in their study are much lower – average of 5.21 kg for an average plot size of 0.33 ha (about 16 kg N/ha, similar to farmer-reported application rates in our household survey), while soil carbon is measured as percent by weight as determined by lab analyses.

[9]Agronomic N use efficiency is calculated for the sample averages according to the following formula: N use efficiency = (maize yield on treatment plots - maize yield on control plots)/total N applied.

[10]Our data corroborate this finding. The average maize yield following the addition of organic resources and nitrogen fertilizer is 4.52 Mg/ha as opposed to 3.33 Mg/ha following the addition of organic resources alone; while average total soil carbon stock is lower: 43.69 Mg/ha vs. 48.68 Mg/ha. These differences are also statistically significant (with the p-value=0.0000).

[11]To account for fixed effects in the estimation of the production function, we add the average of coefficients for each of the fixed effects category (farm, year, treatment, year-treatment) to the coefficient on the constant term: -1.460+0.650=-0.810.

6.2 Soil carbon equation

Annual changes in soil carbon stock reflect the balance between carbon outflows and inflows. Outflows are losses through gas fluxes associated with microbial and plant respiration, water and wind erosion, and deep leaching. Inflows include carbon in crop residues, animal manure, compost, and other organic resources (Blanco-Canqui et al. 2013). As a first-step approximation, we model the annual change in soil carbon, $\Delta c = c_{t+1} - c_t$, as a sum of carbon losses in the form of carbon mineralization and carbon additions in the form of maize residues left on the field for soil fertility management:

$$c_{t+1} - c_t = -Dc_t + A(\alpha_t M)^{\beta}, \tag{13}$$

where D is rate of annual soil carbon loss, A and β are parameters calibrated using the Rothamsted Carbon Model for turnover of carbon in soil (Coleman and Jenkinson 1999), and M is total residues.

According to the Intergovernmental Panel on Climate Change (IPCC) guidelines, the relative soil carbon stock change factor is 0.91 ($\pm$4%) for tropical wet soils with low residue return due to their removal, which implies an average 10 percent annual decrease in soil carbon stock (IPCC 2003). The main loss of soil carbon is carbon dioxide (CO_2) release from the soil surface, referred to as carbon mineralization. This process is mainly a result of microbial decomposition of soil organic matter. While soil organic matter (SOM) is crucial for maintaining overall soil fertility, higher levels of SOM induce greater microbial decomposition rates, leading to higher rates of carbon loss through carbon dioxide mineralization. Using a laboratory experiment to study the impacts of pre-existing SOM on soil mineralization after addition of organic matter in soils from the chronosequence farms, Kimetu et al. (2009) show that carbon losses are greater in the carbon-rich soils than in carbon-poor soils regardless of the quality of the applied organic resource. Total CO_2-C annual mineralization (C loss to C stock ratio) is found to depend on the time in continuous cultivation: it is between 8 and 12 percent over the course of one year. D is assumed to be 0.11.

The main source of soil carbon inputs is maize roots left in the soil and maize stover (leaves and stems) from previous seasons. Parameters A, β and M are chosen to fit the equilibrium levels of soil carbon, shown by the Rothamsted Carbon Model for turnover of carbon in soil (Coleman and Jenkinson 1999), calibrated for the geographic location of the chronosequence farms, as well as

the equilibrium levels of maize yields.[12] Then, the soil carbon equation can be written as follows:

$$c_{t+1} - c_t = g(c_t, \mathbf{x}_t) = -0.11c_t + 4.45(\alpha_t \times 1)^{0.79}. \tag{14}$$

6.3 Prices

We now describe economic variables used in the simulation model: prices of maize grain (p) and nitrogen (n), the per-hectare cost of preparing the land, planting, and harvesting maize (m), opportunity cost of maize residues (q), and the discount rate (δ). Socio-economic farm-level data to derive the economic variables come from the detailed household and production survey in the Nyando and Yala river basins of western Kenya in 2011-2012, described in Berazneva et al. (2014). Price, cost and market data were collected from public sources and interviews with farmers, as well as market sellers and buyers in 2011, 2012, and 2013.

The empirical distributions of prices are summarized in Table 3. We use the average prices as reported in household surveys to reflect small quantity premiums, travel costs, local availability, and other potential transaction costs. The average price of maize grain (p) is 356 \$/Mg, while the average maize price reported in market surveys is slightly higher, 410 \$/Mg. The main sources of nitrogen in western Kenya are found in the fertilizer mixes: Di-ammonium phosphate (DAP) with a nitrogen content of 18 percent is commonly applied during planting, while urea (N content 46 percent) and calcium ammonium nitrate (CAN) (N content 26 percent) are applied as top dressing. The cheapest source of nitrogen is urea fertilizer (2,070 \$/Mg); all three fertilizer types, however, are commonly applied. To represent local availability and preferences, similar to Sheahan, Ariga, and Jayne (2013), we construct a composite price of nitrogen from the prices of the main fertilizer types, using their relative quantities of nitrogen applied on maize fields as weights.[13] The composite price of nitrogen (n) is 4,337 \$/Mg as reported in household surveys (its local market equivalent is 4,390 \$/Mg).[14]

The per-hectare cost of preparing the land, planting, and harvesting maize (m) is derived

[12] The procedure used to calibrate the Rothamsted Carbon Model and estimate the equilibrium levels of soil carbon in the research area is developed by Dominic Woolf. This procedure and the choice of A, β and M are described in Appendix A.2.

[13] The formula used is the following: n=price of DAP/0.18 $\times$ 0.69 + price of urea/0.46 $\times$ 0.16 + price of CAN /0.26 $\times$ 0.15. The weights (0.69 for DAP, 0.16 for urea and 0.15 for CAN) are derived from the household survey.

[14] For comparison, the 2011-2012 average international prices for maize are about 270 \$/Mg and 630 \$/Mg for nitrogen (USDA; U.S. Geological Survey, Mineral Commodity Summaries).

from the household survey and includes the additional monetary and opportunity costs incurred during maize production – the cost of seeds, transportation, equipment, sacks for storage, etc. as well as paid and household labor and land rental value. The opportunity cost of household labor is calculated by multiplying the number of days worked by household members and an average agricultural daily wage of 100 Kenyan shillings. To account for the opportunity cost of land, we run a hedonic analysis of land characteristics using the reported land rental value for the households that rented in or rented out parcels during the household survey. Parcel characteristics include perceived soil type and quality, as well as measured soil carbon content and parcel altitude. We then use the estimated coefficients to calculate the average land rental value in the entire sample of surveyed households.[15]

The use of maize residues for soil fertility management among western Kenyan farmers is traded off against two other competing uses of biomass: household energy and livestock feed. The household survey shows that currently about a quarter of maize residues is used as fuel, another quarter is allocated for feeding animals, and the remaining half is left on the field, mulched, or collected to apply as organic soil amendments. Berazneva et al. (2014) estimate the value of maize residues left on the fields for soil fertility management, using the same household survey. The average value (q) is 72 \$/Mg and it is similar to the value of crop residues in other household uses. While very few households in the survey purchased livestock feed, many bought fuelwood. Specific energy, energy per unit mass that is often used for comparing fuels, of mixed fuel and maize stover and cobs in western Kenya is very similar (Torres-Rojas et al. 2011).[16] The mean (median) price of fuelwood in the household survey is 77 (61) \$/Mg.

Another critical factor affecting farmers' investment in soil resource conservation is the extent to which they discount the future. Higher discount rates lead to lower than optimal steady-state stocks of renewable resources and faster depletion rates of non-renewable resources (Hotelling 1931; Clark 1990). Previous empirical research suggests that the discount rates implied by behavior in field studies or in experimental settings exceed market interest rates, yet they show significant variability in the estimates and suffer from numerous challenges that tend to bias the estimates upward

[15]The hedonic regression and the distribution of all prices used in the analysis are reported in Appendix A.3. While we recognize the difference between land rental value and the longer-term value of owned land, as well as other limitations of the hedonic analysis (Palmquist 2005), it is not the primary focus of our analysis. We use this strategy to approximate average opportunity cost of land in the area.

[16]It is 17.2 MJ/kg for mixed wood, 17.3 MJ/kg for maize stover and 16.9 MJ/kg for maize cobs.

(Frederick, Loewenstein, and O'Donoghue 2002). While there are fewer studies in developing countries, the ones that exist point to additional challenges of constrained credit markets and their implications for discounting and borrowing (Pender 1996). To approximate smallholders' discount rates in western Kenya we surveyed lending institutions – banks, micro-finance institutions, market traders, etc. – in the research area. The survey also showed significant variability: the annual interest rate ranges from 3 to 24 percent, depending on the type of loan, amount, and lending institution. We use the discount rate of ten percent and check the results for the discount rates of five and fifteen percent.

All prices are quoted in US dollars using the 2011-2012 average exchange rate of 84 Kenyan Shillings (KES) per 1 US Dollar. Values for the agronomic and economic variables and prices used in the model together with their sources are summarized in Table 4.

6.4 Difference in resource endowments: three soil fertility levels

The wealth of a household can also be measured by its natural resource endowment. Farms with better soil fertility enjoy higher maize yields that may translate to higher profits and assets. Moreover, richer households are found to be more patient (implying lower discount rates) (Pender 1996; Tanaka, Camerer, and Nguyen 2010). To account for the difference in resource endowments, we start with different initial soil carbon stocks: c_0=20, 33, and 55 Mg/ha, corresponding to farms with depleted, medium-fertility, and fertile soils. c_0=33 and 55 Mg C/ha are the average values of soil carbon stocks on the control plots of the older (converted from forest to agricultural land before 2000) and young farms (converted since 2000) in the chronosequence experiment, respectively. c_0=20 Mg C/ha is chosen to represent degraded soils, and is the average soil carbon stock level in the three household survey villages closest to the chronosequence sites.

7 Results and discussion

The model described in Sections 5 and parametrized in Section 6 maximizes the discounted net present value of maize production over a 25-year horizon, which is defined as the sum of the discounted net revenue from maize production over the interval $t = 0, 1, ..., T - 1$ and a final

function in period $t = T$.[17] To approximate the asymptotic approach to a bioeconomic optimum in a finite-horizon problem, our final function, $\Psi(c_T) = \rho^{T-1}(py(c_T, f_T) - nf_T - qr_T\alpha_T - m)/\delta$, represents the discounted value of maintaining c_T forever by producing y_T forever, beginning in $t = T$.[18] The three initial values of soil carbon, $c_0 = 20$, $c_0 = 33$ and $c_0 = 55$ Mg/ha, correspond to the three levels of resource endowment – degraded, medium-fertility and fertile soils. We assume that the quantity of crop residues is constant across the periods and farms with different initial values of soil carbon: $r_t = ky_t = r = 5.51$ Mg/ha/year.[19]

The solution of our bioeconomic model results in the optimal decision rules for the two management variables – nitrogen input (f_t) and share of residues (α_t) – and the associated values of soil carbon (c_t) and maize yield (y_t). Prior to solving the finite-horizon problem, with final function, we determine the steady state of the infinite-horizon problem which can be compared to the terminal carbon stock in the finite-horizon problem, c_{25}. Sensitivity analysis of both the infinite-horizon steady state and the approach and terminal carbon stock in the finite-horizon problem can then be conducted.

7.1 Steady-state analysis

We first look at the steady-state equilibrium to answer the question whether the optimal management strategies would ever be sustainable *ad infinitum*. For the steady-state equilibrium to exist, we need $f_t = f > 0$ and $\alpha_t = \alpha > 0$. Then, assuming $r_t = ky_t = r$, in steady state $c_{t+1} = c_t = c$, implying the following first order conditions:

$$c = (A/D)(\alpha M)^{\beta}, \tag{15}$$

$$p[\gamma_f + 2\gamma_{ff}f + \gamma_{cf}c] = n, \tag{16}$$

$$\frac{p[\gamma_c + 2\gamma_{cc}c + \gamma_{cf}f][A\beta M(\alpha M)^{\beta-1}]}{(\delta + D)} = qr. \tag{17}$$

It can be shown that c_{ss} will be locally stable if and only if $|\theta'(c_{ss})| < 1$, where $\theta(c_t; f_t, \alpha_t) \equiv (1-D)c_t + A(\alpha_t M)^{\beta}$ (Conrad 2010). Given the parameters in Table 4, we can solve Equations 15, 16 and 17 to get the endogenously determined equilibrium values for soil carbon stock, rate of nitrogen

[17] Changing the horizon length to, for example, T=50 does not change the results.

[18] $\Psi(c_T) = \sum_{t=T}^{\infty} \rho^t(py(c_T, f_T) - nf_T - qr_T\alpha_T - m) = \rho^{T-1}(py(c_T, f_T) - nf_T - qr_T\alpha_T - m)/\delta$.

[19] We are working on relaxing this assumption. The results will be ready for the paper's next draft.

application and share of maize residues to leave on the field for soil fertility management. Table 5 shows the results. For $\delta = 10\%$, $\alpha_{ss} = 0.86$, $f_{ss} = 0.08$ Mg/ha, $c_{ss} = 36.03$ Mg/ha, and y_{ss}=4.05 Mg/ha. The equilibrium value of soil carbon stock, $c_{ss} = 36.03$ Mg/ha, is the same as the long-term equilibrium value of soil carbon obtained with the Rothamsted Carbon Model. $f_{ss} = 0.08$ Mg/ha and $\alpha_{ss} = 0.86$ are both much higher than the current farming practices in western Kenya, which average about 0.014 Mg/ha of nitrogen fertilizer and 53 percent of crop residues applied as organic amendments. Furthermore, the steady-state values imply the equilibrium maize yield y_{ss}=4.05 Mg/ha, which is more than double the average yield reported in the household survey.

The steady-state analysis suggests that the optimal management strategies can be achieved and will be sustainable, leading to high levels of soil carbon stock and associated increases in maize production. These levels of soil carbon and maize yield are, however, dependent on the discount rate (Table 5). A lower discount rate ($\delta = 5\%$) results in a higher steady-state stock of soil carbon and maize yield ($c_{ss} = 40.45$ Mg/ha and $y_{ss} = 4.19$ Mg/ha, respectively). This is consistent with other empirical studies that demonstrate the inverse relationship between discount rates and household profits and assets (Pender 1996; Tanaka, Camerer, and Nguyen 2010).

7.2 Value of carbon

In steady state the shadow price of soil carbon is given by

$$\lambda = \frac{p(\gamma_c + 2\gamma_{cc}c + \gamma_{cf}f)}{1 - \rho(1 - D)}. \tag{18}$$

With $\delta = 10\%$, $c_{ss} = 36.03$ Mg/ha and $f_{ss} = 0.08$ Mg/ha, Equation 18 implies λ_{ss}= 120 \$/Mg (Table 5). The numerator is the net marginal benefit of an additional unit of soil carbon in maize production and is equal to 23.36 \$/Mg. The denominator is the modified discount rate that increases the marginal benefit more than fivefold to show the present value of one Mg of soil carbon when maintained for the rest of time.

Our steady-state shadow price of soil carbon, 120 \$/Mg of carbon or 33 \$/Mg of carbon dioxide equivalent (\$/Mg of CO_2e),[20] is substantially higher than the majority of the existing national and sub-national carbon pricing instruments. Carbon dioxide prices in the European Union Emissions

[20]Conversion from carbon to carbon dioxide is done by multiplying the amount of carbon by 3.667 (WB 2012).

Trading System (EU ETS) remained in the range of 5-9 \$/Mg of CO_2e in 2013 (WB 2014), and the average price for forestry offsets in 2012 was 7.8 \$/Mg of CO_2e (Peters-Stanley, Gonzalez, and Yin 2013). At the same time, our price is closer to the global shadow cost of carbon based on the estimates of future climate change damages. Two recent estimates of an appropriate shadow price of carbon or social cost of carbon are 35.8 \$/Mg of CO_2e for the preferred 3 percent constant discount rate (IWG 2010, 2013) and 18.6 \$/Mg of CO_2e for declining discount rates (Nordhaus 2014). Our estimates highlight the considerable local benefits in the form of soil fertility improvements and increased maize yields, and suggest an alternative method for estimating carbon value.

7.3 Current practices vs. optimal decision rules

We start by running the model with the values for f and α that approximate the current practices of farmers in western Kenya. $f = 0.014$ Mg/ha and $\alpha = 0.53$ are the average values in the household survey. This simulation is instructive for the calibration of the model and to observe the change in soil carbon if current practices are preserved. Not surprisingly the soil carbon stock rapidly declines from the initial levels to the steady state of 24-26 Mg/ha and the corresponding maize yield of 1.97-2.17 Mg/ha (see Figure 3). Similar carbon stocks and maize yields are observed in the household survey. The average values are 20.33 Mg/ha for soil carbon stocks and 1.68 Mg/ha for maize yields in the three villages closest to the chronosequence farms.

We then allow the model to maximize the discounted profit over 25 years with $\delta = 10\%$. Soil carbon stocks reach the steady-state level ($c_{ss} = 36.03$ Mg/ha) for the three farm types within the planning horizon (Table 6, Figure 5). For farms with depleted and medium-fertility soils, soil carbon stock increases; for farms with fertile soils, however, it rapidly declines from $c_0 = 55$ Mg/ha to $c_{ss} = 36.03$ Mg/ha, with the largest decrease in the first ten years. This is consistent with the results of Kinyangi (2008) from the same sites. The soils of the Kakamega-Nandi tropical highland forests accumulated large carbon and nutrient stocks due to a long-term surface litter deposition under natural forest conditions. Following land conversion from forests to agricultural land, Kinyangi (2008) finds significant loss of soil carbon pool during the first 11 years of continuous maize cultivation even with additions of mineral fertilizer. On a global scale, Davidson and Ackerman (1993) find that between 20 and 40 percent of soil carbon is lost following conversion to agriculture in various ecosystems worldwide, with most of this loss occurring within

the first few years following conversion.

The potential for soil carbon sequestration is highest for farms with depleted soils. Over 25 years, the soil carbon stock increases by 15.75 Mg/ha, with an average annual increase of 630 kg/ha of carbon. And for farms with medium soil fertility the average annual increase is 117 kg/ha. These two numbers are the upper and lower bounds on the average annual carbon sequestration rate of 375 kg/ha found from the use of crop residues for soil fertility management across the 46 studies in Sub-Saharan Africa (WB 2012).[21] And this average rate is similar to the rates from applying mulches, planting cover crops, and practicing no-tillage.

Reaching and sustaining the steady-state values of soil carbon and corresponding maize yield requires different initial management strategies for the farms with different soil fertility (Figure 4). For farms with depleted and medium-fertility soils, the optimal rate of nitrogen input and share of residues are high from the beginning: $f_0 = 0.12$ Mg/ha and $\alpha_0 = 1$ for farms with depleted soils and $f_0 = 0.09$ and $\alpha_0 = 0.89$ for farms with medium-soil fertility. Maintaining high maize yields on farms with fertile soils is, however, initially possible with much lower rates: $f_0 = 0.03$ Mg/ha and $\alpha_0 = 0.71$. As soil carbon stock declines, higher applications rates are required (see Table 6).

The equilibrium maize yield due to the optimal decision rules in our model is similar to the average yields from the researcher-managed chronosequence plots. Using the data from the nearby Vihiga, Kakamega and Teso districts and simulations of the soil-crop dynamic model of nutrient balances (DYNBAL), Tittonell et al. (2006, 2007) also find that maize grain yields increase with increasing contents of soil carbon and nitrogen, with the potential maize grain yields varying between 10.8 and 11.4 Mg/ha. Their yields are much higher than the yields shown by our model. In addition to the soil-crop interactions, we also consider the socio-economic constraints, such as high prices of external inputs, competing uses for maize residues, and farmers' time preferences. Our results, hence, are more reflective of both biophysical and socio-economic constraints on production.

The difference in the present value of profit from a representative hectare of land is also observed between the farms with different initial soil fertility (Table 6). The profit is greatest for the farms with the best initial conditions (5,332 \$/ha) – farms with high starting values of soil carbon

[21] The World Bank's calculation of the average carbon sequestration rate is based on estimating the cost-effectiveness of the land management practices, assuming the discount rate of 9 percent and the adoption period of 25 years, the time period soil carbon sequestration reaches saturation for most of the land management technologies (WB 2012). Our discount rate and time period are comparable.

stock. This finding highlights the importance of initial natural resource base for maize yields and consequent farmer livelihoods: high initial carbon stocks allow for maintaining soil fertility over time with lower initial rates of costly external inputs, while low initial carbon stocks require substantial fertilizer applications from the start, in the range of 90-130 kg/ha, and in their absence may give rise to "soil degradation poverty traps" (Marenya and Barrett 2009b; Stephens et al. 2012).

7.4 Sensitivity analysis: changing prices

The price of maize grain, nitrogen and maize residues used are the mean values of the respective empirical distributions of prices reported in the household survey and estimated in Berazneva et al. (2014) (Table 3 shows their summary statistics). They are subject to small quantity premiums, travel costs, local availability, and other potential transaction costs. To explore the sensitivity of our results to the changes in prices, we use additional information from the empirical distributions. Table 7 shows the sensitivity of the steady-state values of α_{ss} and c_{ss}, when other values of p, n and q are used (assuming $\delta = 10\%$). We simultaneously change the values of p, n and q to their 25th percentile, 50th percentile, and 75th percentile in the distribution, as well as decrease the average values by subtracting or adding 50 or 25 percent of the respective standard deviations ($\mu - 0.5\sigma$, $\mu - 0.25\sigma$, $\mu + 0.25\sigma$, $\mu + 0.5\sigma$). Our results hold up to the price values in the range of $\mu - 0.25\sigma$ to $\mu + 0.25\sigma$. When one or two of the prices are changed (increasing or decreasing by 25% of the standard deviation), our results are the most sensitive to the value of q (Table 8).[22] This is not surprising given the uncertainty in estimating the opportunity cost of maize residues.

8 Conclusion

We examine the link between natural resources and agricultural production in smallholder systems of western Kenya. Our findings show that regardless of the initial soil fertility levels it is agronomically possible to double maize yields and increase and maintain large stocks of soil carbon. The optimal management strategies lead to maize yields of 3.5-4.2 Mg/ha and carbon stocks of 20.2-40.2 Mg/ha (in the top 0.1 m), with discount rates of five to fifteen percent and given the prevailing

[22] The values of α_{ss} and c_{ss} stay within the expected range, as long as q is kept constant and either p, n or p and n are decreased by 25 percent of the respective standard deviation, or if either of the three prices or two of them are increased while the other two or one are kept constant. Only when we increase p and n by 25 percent of the standard deviation and keep q at its mean value, the steady-state value of α_{ss} hits its upper bound ($0 \leq \alpha \leq 1$).

price levels. Reaching and sustaining these high maize yields and soil carbon stocks, however, requires considerable application rates of mineral fertilizer and organic resources – far higher than the current practices of western Kenya farmers. Combining applications of both mineral fertilizer and organic resources takes advantage of the positive effects on soil quality, and this has future benefits in the form of increased productivity. Application rates also depend on the initial condition of soil fertility. For farms with depleted and medium-fertility soils, the optimal rates of nitrogen input and share of residues are high from the beginning (0.09-0.12 Mg/ha and 0.89-1, respectively), while fertile soils permit lower applications rates initially. Better initial natural resource endowment (more fertile soils) also translates to higher profits: the discounted net present value of agricultural profits from a representative hectare of land is over US$5,000 for farms with fertile soils, and reduces to less than $3,000 and $1,500 for farms with medium-fertility and degraded soils, respectively.

Our findings are also sensitive to the discount rate and the values of prices. For example, the equilibrium value of soil carbon stock is the highest with the lower discount rate (40.45 Mg/ha for the discount rate of 5 percent) and decreases as the discount rate goes up (36.03 Mg/ha for the discount rate of 10 percent and 20.19 Mg/ha for the discount rate of 15 percent). Lower rates of time preference imply higher application rates of organic resources and mineral fertilizer. The resulting increase in maize productivity translates into future financial benefits that are accounted for in the dynamic model. High rates of time preference, on the other hand, result in lower application rates, lower yields, and agricultural profits, thus perpetuating soil degradation and poverty.

The high application rates of maize residues can be achieved in smallholder systems, however, only if alternative sources for competing uses are identified and made available. Removing crop residues for fodder and fuel are prevailing practices throughout the developing world in the tropics and subtropics (Lal 2006). In the absence of readily available and affordable substitutes, removing crop residues from agricultural fields contributes to depletion of soil fertility, thus decreasing agronomic productivity and reducing fertilizer efficiency. Sustainable management of soil resources that are fundamental in agrarian societies, therefore, needs to become part of any government policy or program aimed at improving agricultural productivity, achieving food security, and eliminating poverty. Such strategies could include fertilizer subsidies, extension services, establishment of biofuel plantations on degraded and marginal lands, and improving access to credit.

This analysis also has implications for global climate policy debates in terms of understanding

the potential of soil carbon sequestration in mitigating climate change. Our findings show that in the western Kenyan highlands considerable amounts of carbon can be sequestered over 25 years. The soil carbon stock changes for depleted and medium-fertility soils equate to an average annual increase in soil carbon of 630 and 117 kg C/ha, respectively. Our findings also show high equilibrium on-farm value of soil carbon: it ranges between 108 and 148 US$/Mg of carbon or 29 and 40 US$/Mg of carbon dioxide equivalent, depending on the discount rate used. These estimates could be higher if off-farm benefits were to be included. Our estimates highlight the considerable local benefits of carbon in the form of soil fertility improvements and increased maize yields, and suggest an alternative method for estimating carbon value.

In this paper we consider profit maximization as the only objective of farming households and soil carbon as the state variable. To account for other objectives of farming households and the existing socio-economic and biophysical constraints, the objective function could be further extended to incorporate, for example, the objective of food security; additional constraints and farming practices can also be included. The model set-up also allows us to introduce participation in a Clean Development Mechanism project to receive payments for carbon sequestration services, as in Wise and Cacho (2011). The income provided by carbon payments could partially counteract the effects of high discount rates and we expect in this scenario it would be possible to achieve even higher soil carbon stocks and corresponding maize yields.

References

Antle, J., and J. Stoorvogel. 2008. "Agricultural carbon sequestration, poverty, and sustainability." *Environment and Development Economics* 13:327–352.

Barbier, B. 1998. "Induced innovation and land degradation: Results from a bioeconomic model of a village in West Africa." *Agricultural Economics* 19:15–25.

Barbier, B., and G. Bergeron. 2001. "Natural resource management in the hillsides of Honduras: bioeconomic modeling at the micro-watershed level." Research Report No. 123, International Food Policy Research Institute (IFPRI), Washington, D.C.

Berazneva, J., D.R. Lee, F. Place, and G. Jakubson. 2014. "Allocation and valuation of non-marketed crop residues in smallholder agriculture: the case of maize residues in western Kenya." Working paper, Cornell University, Ithaca, NY.

Berck, P., and G. Helfand. 1990. "Reconciling the von Liebig and differentiable crop production functions." *American Journal of Agricultural Economics* 72:985–996.

Betemariam, E., T.G. Vagen, K. Shepherd, and L. Winowiecki. 2011. "A Protocol for Measurement and Monitoring Soil Carbon Stocks in Agricultural Landscapes." Version 1.1, World Agroforestry Centre (ICRAF), Nairobi, Kenya.

Blanco-Canqui, H., C.A. Shapiro, C.S. Wortmann, R.A. Drijber, M. Mamo, T.M. Shaver, and R.B. Ferguson. 2013. "Soil organic carbon: The value to soil properties." *Journal of Soil and Water Conservation* 68:129A–134A.

Burt, O.R. 1981. "Farm Level Economics of Soil Conservation in the Palouse Area of the Northwest." *American Journal of Agricultural Economics* 63:83 –92.

Chivenge, P., B. Vanlauwe, and J. Six. 2011. "Does the combined application of organic and mineral nutrient sources influence maize productivity? A meta-analysis." *Plant and Soil* 342:1–30.

Chivenge, P.P., H.K. Murwira, K.E. Giller, P. Mapfumo, and J. Six. 2007. "Long-term impact of reduced tillage and residue management on soil carbon stabilization: Implications for conservation agriculture on contrasting soils." *Soil and Tillage Research* 94:328–337.

Clark, C.W. 1990. *Mathematical bioeconomics: the optimal management of renewable resources*, 2nd ed. Pure and applied mathematics, New York: John Wiley & Sons.

Coleman, K., and D. Jenkinson. 1999. "ROTHC-26.3. A model for the turnover of carbon in soil." November 1999 issue (modified august 2008), Rothamsted Research, Harpenden, Herts, UK.

Conrad, J. 2010. *Resource economics*, 2nd ed. New York, NY: Cambridge University Press.

Crowley, E., and S. Carter. 2000. "Agrarian Change and the Changing Relationships Between Toil and Soil in Maragoli, Western Kenya (1900–1994)." *Human Ecology* 28:383–414.

Dasgupta, P. 2010. "The Place of Nature in Economic Development." In D. Rodrik and M. Rosenzweig, eds. *Handbook of Development Economics*. Elsevier, vol. 5 of *Handbooks in Economics*, pp. 4977–5046.

Davidson, E., and I. Ackerman. 1993. "Changes in Soil Carbon Inventories Following Cultivation of Previously Untilled Soils." *Biogeochemistry* 20:161–193.

de Janvry, A., M. Fafchamps, and E. Sadoulet. 1991. "Peasant Household Behaviour with Missing Markets: Some Paradoxes Explained." *Economic Journal* 101:1400–1417.

Diaz-Zorita, M., G.A. Duarte, and J.H. Grove. 2002. "A review of no-till systems and soil management for sustainable crop production in the subhumid and semiarid Pampas of Argentina." *Soil and Tillage Research* 65:1–18.

Duflo, E., M. Kremer, and J. Robinson. 2011. "Nudging Farmers to Use Fertilizer: Theory and Experimental Evidence from Kenya." *American Economic Review* 101:2350–2390.

FAO. 2005. "New_LocClim: Local Climate Estimator."

Frederick, S., G. Loewenstein, and T. O'Donoghue. 2002. "Time Discounting and Time Preference: A Critical Review." *Journal of Economic Literature* 40:351–401.

French, D. 1986. "Confronting an unsolvable problem: Deforestation in Malawi." *World Development* 14:531–540.

Gavian, S., and M. Fafchamps. 1996. "Land tenure and allocative efficiency in Niger." *American Journal of Agricultural Economics* 78:460–471.

Gentile, R., B. Vanlauwe, P. Chivenge, and J. Six. 2011. "Trade-offs between the short- and long-term effects of residue quality on soil C and N dynamics." *Plant and Soil* 338:159–169.

Goldstein, M., and C. Udry. 2008. "The Profits of Power: Land Rights and Agricultural Investment in Ghana." *Journal of Political Economy* 116:981–1022.

Graff-Zivin, J., and L. Lipper. 2008. "Poverty, Risk, and the Supply of Soil Carbon Sequestration." *Environment and Development Economics* 13:353–373.

Guerena, D. 2014. "Anthropogenic and natural pyrogenic carbon in tropical environments: from the rhizosphere to the landscape." PhD dissertation, Cornell University, Ithaca, NY.

Harou, A., Y. Liu, C.B. Barrett, and L. You. 2014. "Spatiotemporally Heterogeneous Expected Returns to Fertilzer and Poverty Rates in Malawi." Working paper, Cornell University, Ithaca, NY.

Hassan, R., R. Scholes, and N. Ash, eds. 2005. *Ecosystems and Human Well-being: Current State and Trends, Volume 1*. Washington, DC: Island Press.

Holden, S., and H. Binswanger. 1998. "Small farmers, market imperfections, and natural resource management." In E. Lutz, H. Binswanger, P. Hazell, and A. McCalla, eds. *Agriculture and the Environment: Perspectives on Sustainable Rural Development*. Washington, D.C.: The World Bank (WB), pp. 50–70.

Holden, S., B. Shiferaw, and J. Pender. 2005. "Policy analysis for sustainable land management and food security in Ethiopia: A bioeconomic model with market imperfections." Research Report No. 140, International Food Policy Research Institute (IFPRI), Washington, DC.

Hotelling, H. 1931. "The Economics of Exhaustible Resources." *Journal of Political Economy* 39:137–175.

IPCC. 2003. *Intergovernmental Panel on Climate Change Good Practice Guidance for Land Use, Land-Use Change and Forestry*. Institute for Global Environmental Strategies (IGES) for the Intergovernmental Panel on Climate Change (IPCC).

IWG. 2010. *Interagency Working Group on Social Cost of Carbon, US Government, technical support document: Social cost of carbon for regulatory impact analysis under Executive Order 12866*. Interagency Working Group (IWG).

—. 2013. *Interagency Working Group on Social Cost of Carbon, US Government, technical support document: Technical update of the social cost of carbon for regulatory impact analysis under Executive Order 12866, May 2013*. Interagency Working Group (IWG).

Khan, S.A., R.L. Mulvaney, T.R. Ellsworth, and C.W. Boast. 2007. "The Myth of Nitrogen Fertilization for Soil Carbon Sequestration." *Journal of Environment Quality* 36:1821.

Kimetu, J., J. Lehmann, J. Kinyangi, C. Cheng, J. Thies, D. Mugendi, and A. Pell. 2009. "Soil organic C stabilization and thresholds in C saturation." *Soil Biology & Biochemistry* 41:2100–2104.

Kimetu, J., J. Lehmann, S. Ngoze, D. Mugendi, J. Kinyangi, S. Riha, L. Verchot, J. Recha, and A. Pell. 2008. "Reversibility of soil productivity decline with organic matter of differing quality along a degradation gradient." *Ecosystems* 11:726–739.

Kinyangi, J. 2008. "Soil Degradation, Thresholds and Dynamics of Long-Term Cultivation: from Landscape Biogeochemistry to Nanoscale Biogeocomplexity." PhD dissertation, Cornell University, Ithaca, New York.

Knorr, M., S.D. Frey, and P.S. Curtis. 2005. "Nitrogen additions and litter decomposition: a meta-analysis." *Ecology* 86:3252–3257.

Lal, R. 2006. "Enhancing crop yields in the developing countries through restoration of the soil organic carbon pool in agricultural lands." *Land Degradation & Development* 17:197–209.

—. 2004. "Soil Carbon Sequestration Impacts on Global Climate Change and Food Security." *Science* 304:1623 –1627.

Latshaw, W., and E. Miller. 1924. "Elemental Composition of the Corn Plant." *Journal of Agricultural Research* XXVII:845–860.

Lopez, R. 1997. "Environmental externalities in traditional agriculture and the impact of trade liberalization: the case of Ghana." *Journal of Development Economics* 53:17–39.

Marenya, P., and C. Barrett. 2009a. "Soil quality and fertilizer use rates among smallholder farmers in western Kenya." *Agricultural Economics* 40:561–572.

—. 2009b. "State-conditional Fertilizer Yield Response on Western Kenyan Farms." *American Journal of Agricultural Economics* 91:991–1006.

Matsumoto, T., and T. Yamano. 2011. "Optimal Fertilizer Use on Maize Production in East Africa." In T. Yamano, K. Otsuka, and F. Place, eds. *Emerging Development of Agriculture in East Africa*. Springer Netherlands, pp. 117–132.

McConnell, K. 1983. "An Economic Model of Soil Conservation." *American Journal of Agricultural Economics* 65:83.

Murty, D., M. Kirschbaum, R. Mcmurtrie, and H. Mcgilvray. 2002. "Does conversion of forest to agricultural land change soil carbon and nitrogen? A review of the literature." *Global Change Biology* 8:105–123.

Ngoze, S., S. Riha, J. Lehmann, L. Verchot, J. Kinyangi, D. Mbugua, and A. Pell. 2008. "Nutrient constraints to tropical agroecosystem productivity in long-term degrading soils." *Global Change Biology* 14:2810–2822.

Nordhaus, W. 2014. "Estimates of the Social Cost of Carbon: Concepts and Results from the DICE-2013R Model and Alternative Approaches." *Journal of the Association of Environmental and Resource Economists* 1:273–312.

Palmquist, R.B. 2005. "Property Value Models. Chapter 16." In K.-G. Maler and J. R. Vincent, eds. *Handbook of Environmental Economics*. Elsevier, vol. 2 of *Valuing Environmental Changes*, pp. 763–819.

Pender, J.L. 1996. "Discount rates and credit markets: Theory and evidence from rural india." *Journal of Development Economics* 50:257–296.

Peters-Stanley, M., G. Gonzalez, and D. Yin. 2013. *Covering New Ground: State of the Forest Carbon Markets 2013*. Washington, D.C.: Forest Trends' Ecosystem Marketplace.

Potter, C.S. 1999. "Terrestrial Biomass and the Effects of Deforestation on the Global Carbon Cycle Results from a model of primary production using satellite observations." *BioScience* 49:769–778.

Sanchez, P. 2002. "Soil fertility and hunger in Africa." *Science* 295:2019–2020.

Sheahan, M., J. Ariga, and T. Jayne. 2013. "Modeling the Effects of Input Market Reforms on Fertilizer Demand and Maize Production: A Case Study of Kenya." Unpublished.

Sheahan, M., R. Black, and T. Jayne. 2013. "Are Kenyan farmers under-utilizing fertilizer? Implications for input intensification strategies and research." *Food Policy* 41:39–52.

Shepherd, K.D., and M.J. Soule. 1998. "Soil fertility management in west Kenya: dynamic simulation of productivity, profitability and sustainability at different resource endowment levels." *Agriculture, Ecosystems & Environment* 71:131–145.

Smil, V. 1999. "Crop Residues: Agriculture's Largest Harvest." *BioScience* 49:299–308.

Solomon, D., J. Lehmann, J. Kinyangi, W. Amelung, I. Lobe, A. Pell, S. Riha, S. Ngoze, L. Verchot, D. Mbugua, J. Skjemstad, and T. Schfer. 2007. "Long-term impacts of anthropogenic perturbations on dynamics and speciation of organic carbon in tropical forest and subtropical grassland ecosystems." *Global Change Biology* 13:511–530.

Stephens, E.C., C.F. Nicholson, D.R. Brown, D. Parsons, C.B. Barrett, J. Lehmann, D. Mbugua, S. Ngoze, A.N. Pell, and S.J. Riha. 2012. "Modeling the impact of natural resource-based poverty traps on food security in Kenya: The Crops, Livestock and Soils in Smallholder Economic Systems (CLASSES) model." *Food Security* 4:423–439.

Stevens, P.R., and T.W. Walker. 1970. "The Chronosequence Concept and Soil Formation." *The Quarterly Review of Biology* 45:333–350.

Suri, T. 2011. "Selection and Comparative Advantage in Technology Adoption." *Econometrica* 79:159–209.

Tanaka, T., C.F. Camerer, and Q. Nguyen. 2010. "Risk and Time Preferences: Linking Experimental and Household Survey Data from Vietnam." *American Economic Review* 100:557–571.

Teklewold, H. 2012. "The impact of shadow prices and farmers' impatience on the allocation of a multipurpose renewable resource in Ethiopia." *Environment and Development Economics* 17:479–505.

Thampapillai, D., and J. Anderson. 1994. "A Review of the Socio-Economic Analysis of Soil Degradation Problems for Developed and Developing Countries." *Review of Marketing and Agricultural Economics* 62.

Tittonell, P., and K. Giller. 2013. "When yield gaps are poverty traps: The paradigm of ecological intensification in African smallholder agriculture." *Field Crops Research* 143:76–90.

Tittonell, P., P.A. Leffelaar, B. Vanlauwe, M.T. van Wijk, and K.E. Giller. 2006. "Exploring diversity of crop and soil management within smallholder African farms: A dynamic model for simulation of N balances and use efficiencies at field scale." *Agricultural Systems* 91:71–101.

Tittonell, P., B. Vanlauwe, N. de Ridder, and K.E. Giller. 2007. "Heterogeneity of crop productivity and resource use efficiency within smallholder Kenyan farms: Soil fertility gradients or management intensity gradients?" *Agricultural Systems* 94:376–390.

Torres-Rojas, D., J. Lehmann, P. Hobbs, S. Joseph, and H. Neufeldt. 2011. "Biomass availability, energy consumption and biochar production in rural households of Western Kenya." *Biomass and Bioenergy* 35:3537–3546.

UN. 2014. *The Millennium Development Goals Report*. New York, NY: United Nations (UN).

Vanlauwe, B., and K. Giller. 2006. "Popular myths around soil fertility management in sub-Saharan Africa." *Agriculture, Ecosystems & Environment* 116:34–46.

Waithaka, M., P. Thornton, M. Herrero, and K. Shepherd. 2006. "Bio-economic evaluation of farmers' perceptions of viable farms in western Kenya." *Agricultural Systems* 90:243–271.

WB. 2012. *Carbon Sequestration in Agricultural Soils*. REPORT NO. 67395-GLB, Washington, D.C.: The World Bank (WB).

—. 2014. *State and trends of carbon pricing 2014*. REPORT NO. 88284, Washington, D.C.: The World Bank (WB).

Wise, R., and O. Cacho. 2011. "A bioeconomic analysis of the potential of Indonesian agroforests as carbon sinks." *Environmental Science & Policy* 14:451–461.

WRI. 2007. *Natures benefits in Kenya: An atlas of ecosystems and human well-being*. Washington, D.C., Nairobi, Kenya: World Resources Institute (WRI); Department of Resource Surveys and Remote Sensing, Ministry of Environment and Natural Resources, Kenya; Central Bureau of Statistics, Ministry of Planning and National Development, Kenya; International Livestock Research Institute.

9 Figures and Tables

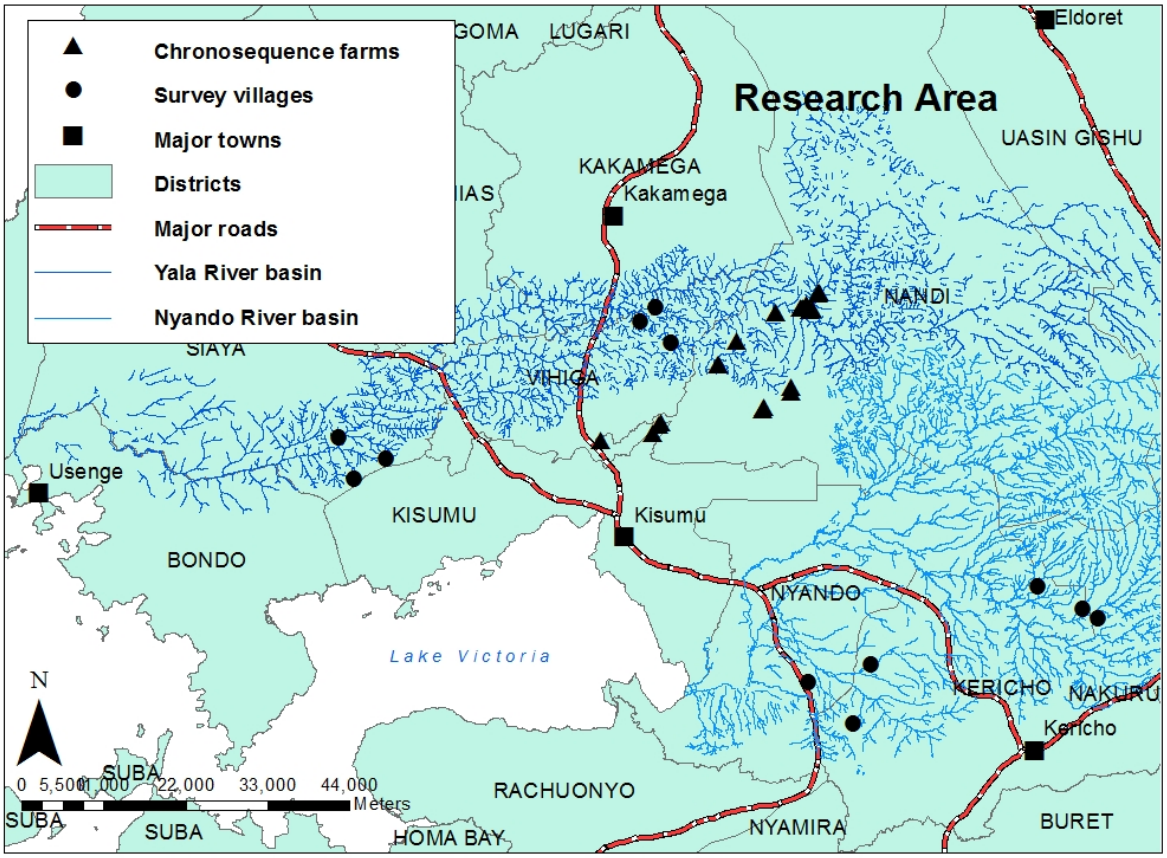

Figure 1: Map of the research farms and villages.

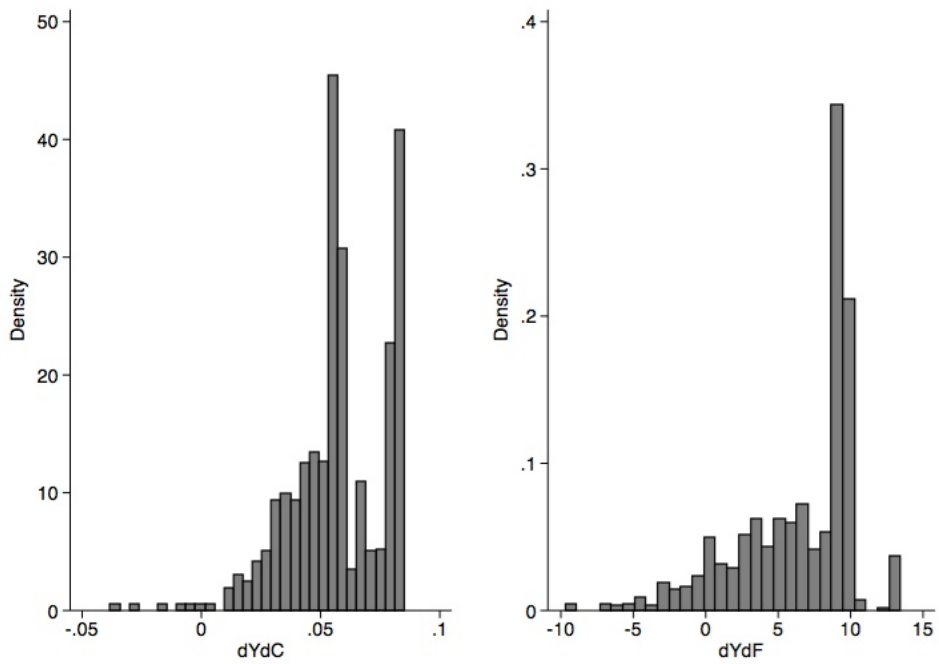

Figure 2: Estimated returns to soil carbon, dYdC, (N=1,450) and nitrogen fertilizer, dYdF, (N=882 observations with $f > 0$).

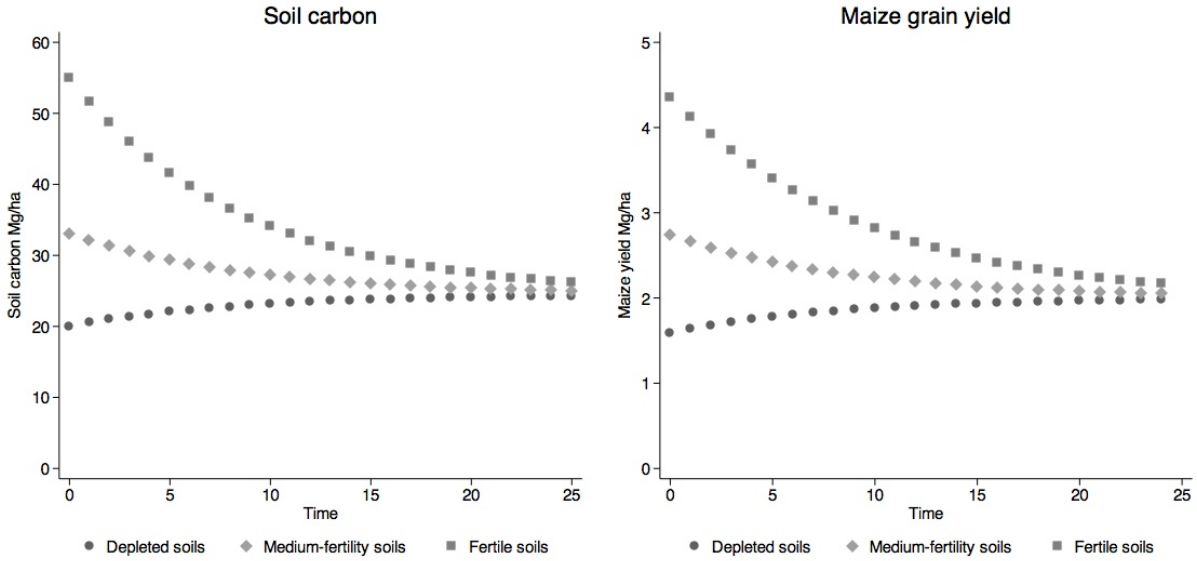

Figure 3: Current practices: f=0.014 Mg/ha and α=0.53 for depleted, medium-fertility and fertile soils.

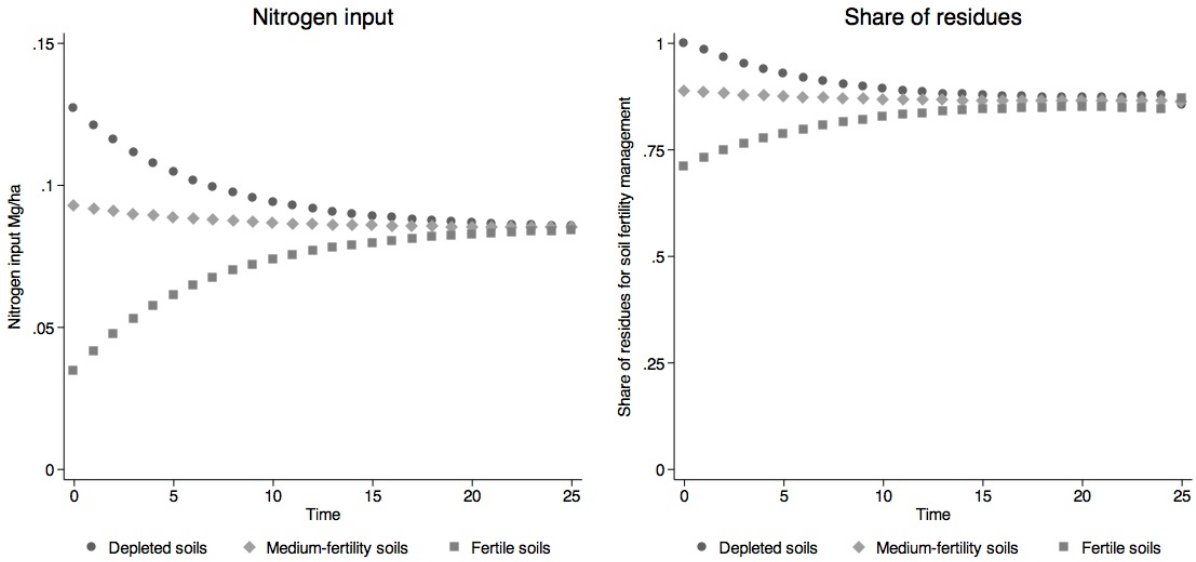

Figure 4: Optimal decision rules: time paths for nitrogen input, f_t, and share of residues for soil fertility management, α_t (δ=10%).

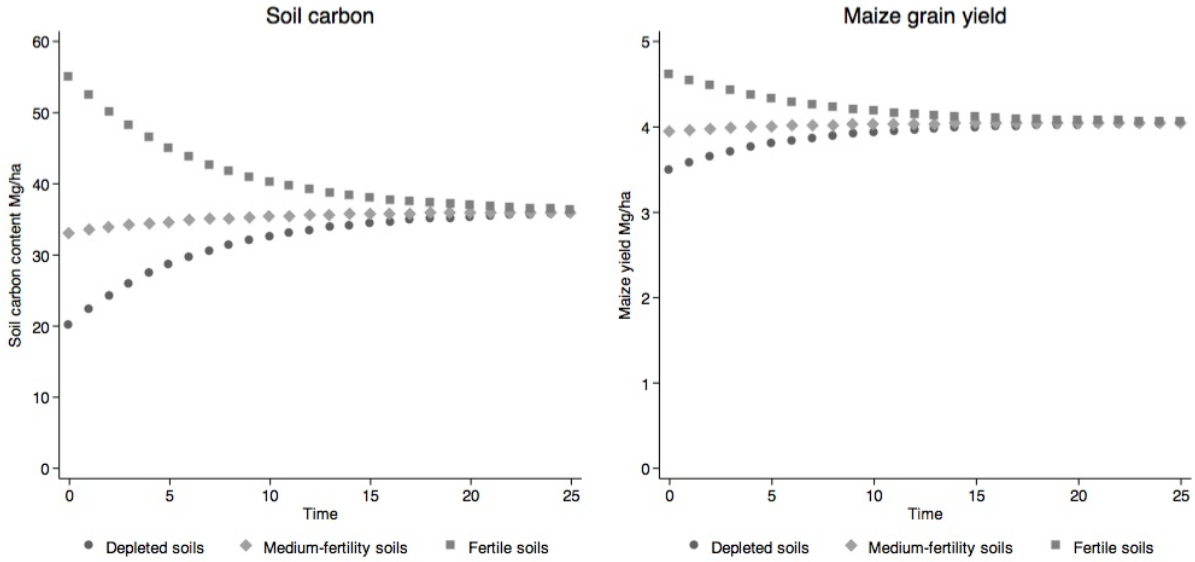

Figure 5: Optimal decision rules: time paths for soil carbon stock, c_t, and maize yield, y_t (δ=10%).

Table 1: Summary statistics for maize production function (N=1,450).

Variable	Unit	Mean	St. deviation	Min	Max
Grain yield	Mg/ha	4.05	2.47	0.16	12.14
Carbon stock	Mg/ha	45.64	18.53	31.79	182.49
Nitrogen fertilizer	Mg/ha	0.08	0.07	0	0.24

Table 2: Maize yield as a function of soil carbon stock and nitrogen fertilizer.

Maize yield (Mg/ha)	(1) Clustered	(2) Bootstrapped
Soil carbon stock (Mg/ha)	0.113***	0.113***
	(0.0354)	(0.0264)
Squared: Soil carbon stock (Mg/ha)	-0.000413**	-0.000413***
	(0.000157)	(0.000126)
Nitrogen fertilizer (Mg/ha)	27.04***	27.04***
	(4.704)	(3.119)
Squared: Nitrogen fertilizer (Mg/ha)	-41.30***	-41.30***
	(10.12)	(10.46)
Interaction: Carbon stock and Nitrogen fertilizer (Mg/ha)	-0.218***	-0.218***
	(0.0632)	(0.0564)
Constant	-1.461	-1.461
	(1.215)	(0.910)
Observations	1,450	1,450
R-squared	0.500	0.500
Farmer, Year, Treatment Fixed Effects	YES	YES
Year-Treatment Interaction	YES	YES
Return to soil carbon stock mean (st.dev.)	0.06 (0.02)	0.06 (0.02)
Return to nitrogen fertilizer mean (st.dev.)	10.39 (6.58)	10.39 (6.58)

*** p<0.01, ** p<0.05, * p<0.1. **Column (1)** shows standard errors clustered at farm level (28 farms). **Column (2)** shows bootstrapped standard errors (1,000 replications).

Table 3: Summary statistics: price of maize, p, nitrogen, n, opportunity cost of residues, q, and per-hectare production cost, m.

Price	Unit	Mean μ	Median 50%	St. dev. σ	25%	75%	N
p	\$/Mg	356	344	99	294	397	100
n	\$/Mg	4,337	4,630	1,624	3,120	5,291	133
q	\$/Mg	72	71	52	31	106	110
m	\$/ha	448	375	303	268	518	247

Table 4: Baseline parameter values for economic and agronomic variables.

Variable	Description	Value	Unit	Source
Maize yield function				
γ_c	Coefficient on c_t	0.113	–	Chronosequence data
γ_{cc}	Coefficient on c_t^2	-0.0004	–	Chronosequence data
γ_f	Coefficient on f_t	27.038	–	Chronosequence data
γ_{ff}	Coefficient on f_t^2	-41.295	–	Chronosequence data
γ_{cf}	Coefficient on $c_t f_t$	-0.218	–	Chronosequence data
γ_0	Constant	-0.810	–	Chronosequence data
Soil carbon equation				
D	Rate of soil carbon loss	0.11	–	IPCC (2003); Kimetu et al. (2009)
A	Carbon plant input parameter	4.45	–	Chronosequence data, ROTHC-26.3
β	Carbon plant input parameter	0.79	–	Chronosequence data, ROTHC-26.3
M	Total residues	1	–	Chronosequence data, ROTHC-26.3
Agronomic parameters				
r	Residues in profit function	5.51	Mg/ha	Chronosequence data
k	Average straw to grain ratio	1.9	–	Chronosequence data
Prices				
p	Price of maize	356	\$/Mg	Market and household surveys
n	Price of nitrogen fertilizer	4,337	\$/Mg	Market and household surveys
q	Value of crop residues	72	\$/Mg	Household survey
m	Maize production cost	448	\$/ha	Household survey
δ	Discount rate	5, 10, 15	%	Market survey
Initial conditions				
c_0	C stock in depleted soils	20	Mg/ha	Chronosequence data
	C stock in medium-fertility soils	33	Mg/ha	Chronosequence data
	C stock in fertile soils	55	Mg/ha	Chronosequence data

Table 5: Steady-state values: changing discount rate.

Variable	Unit	$\delta = 5\%$	$\delta = 10\%$	$\delta = 15\%$
Share of residues, α_{ss}	$0-1$	1	0.86	0.41
Nitrogen input, f_{ss}	Mg/ha	0.07	0.08	0.13
Carbon stock, c_{ss}	Mg/ha	40.45	36.03	20.19
Maize yield, y_{ss}	Mg/ha	4.19	4.05	3.50
Value of carbon, λ_{ss}	\$/Mg	148.23	120.37	107.88

Table 6: Time paths for share of residues, α_t, nitrogen input, f_t, soil carbon stock, c_t, maize yield, y_t, and discounted annual profit, $\rho^t\pi_t$, over 25 cycles for the farms with different resource endowments (δ=10%).

	Depleted soils					**Medium-fertility soils**					**Fertile soils**				
t	α_t	f_t	c_t	y_t	$\rho^t\pi_t$	α_t	f_t	c_t	y_t	$\rho^t\pi_t$	α_t	f_t	c_t	y_t	$\rho^t\pi_t$
0	1.00	0.13	20.00	3.49	-152.07	0.89	0.09	33.00	3.95	202.36	0.71	0.03	55.00	4.61	762.70
1	0.99	0.12	22.25	3.58	-83.17	0.88	0.09	33.42	3.96	194.15	0.73	0.04	52.35	4.54	634.55
2	0.97	0.12	24.20	3.64	-30.75	0.88	0.09	33.78	3.97	184.43	0.75	0.05	50.06	4.47	530.26
3	0.95	0.11	25.87	3.70	6.74	0.88	0.09	34.09	3.98	173.85	0.76	0.05	48.09	4.42	445.20
4	0.94	0.11	27.30	3.75	32.98	0.88	0.09	34.35	3.99	162.86	0.78	0.06	46.39	4.37	375.64
5	0.93	0.10	28.53	3.80	50.80	0.87	0.09	34.58	4.00	151.82	0.79	0.06	44.93	4.32	318.57
6	0.92	0.10	29.59	3.83	62.34	0.87	0.09	34.78	4.00	140.95	0.80	0.06	43.67	4.29	271.57
7	0.91	0.10	30.49	3.86	69.21	0.87	0.09	34.95	4.01	130.42	0.81	0.07	42.59	4.25	232.70
8	0.90	0.10	31.27	3.89	72.67	0.87	0.09	35.09	4.01	120.35	0.81	0.07	41.66	4.22	200.41
9	0.90	0.10	31.93	3.91	73.64	0.87	0.09	35.22	4.02	110.80	0.82	0.07	40.86	4.20	173.45
10	0.89	0.09	32.50	3.93	72.85	0.87	0.09	35.32	4.02	101.81	0.83	0.07	40.17	4.18	150.83
11	0.89	0.09	32.99	3.95	70.81	0.87	0.09	35.41	4.03	93.40	0.83	0.08	39.58	4.16	131.75
12	0.88	0.09	33.42	3.96	67.95	0.87	0.09	35.49	4.03	85.57	0.83	0.08	39.07	4.14	115.57
13	0.88	0.09	33.78	3.97	64.55	0.87	0.09	35.56	4.03	78.31	0.84	0.08	38.63	4.13	101.78
14	0.88	0.09	34.09	3.98	60.85	0.87	0.09	35.62	4.03	71.59	0.84	0.08	38.25	4.12	89.96
15	0.88	0.09	34.36	3.99	57.00	0.86	0.09	35.67	4.03	65.40	0.84	0.08	37.93	4.11	79.77
16	0.88	0.09	34.59	4.00	53.13	0.86	0.09	35.71	4.03	59.70	0.85	0.08	37.65	4.10	70.96
17	0.87	0.09	34.79	4.00	49.31	0.86	0.09	35.75	4.04	54.46	0.85	0.08	37.40	4.09	63.29
18	0.87	0.09	34.97	4.01	45.61	0.86	0.09	35.78	4.04	49.65	0.85	0.08	37.19	4.08	56.59
19	0.87	0.09	35.12	4.02	42.07	0.86	0.09	35.81	4.04	45.25	0.85	0.08	37.01	4.08	50.72
20	0.87	0.09	35.25	4.02	38.71	0.86	0.09	35.84	4.04	41.23	0.85	0.08	36.85	4.07	45.55
21	0.87	0.09	35.37	4.02	35.55	0.86	0.09	35.86	4.04	37.54	0.85	0.08	36.70	4.07	40.98
22	0.87	0.09	35.48	4.03	32.58	0.86	0.09	35.88	4.04	34.18	0.85	0.08	36.57	4.06	36.94
23	0.87	0.09	35.57	4.03	29.81	0.86	0.09	35.90	4.04	31.11	0.85	0.08	36.46	4.06	33.34
24	0.88	0.09	35.66	4.03	27.24	0.86	0.09	35.91	4.04	28.31	0.84	0.08	36.35	4.06	30.15
25	0.86	0.09	35.75	4.04	283.07	0.86	0.08	35.93	4.04	285.06	0.87	0.08	36.24	4.05	288.51
	Total net revenue				\$1,133.46	**Total net revenue**				\$2,734.57	**Total net revenue**				\$5,331.74

Table 7: Sensitivity analysis: steady-state values for share of residues, α_{ss}, and soil carbon, c_{ss}, when changing all prices to the same extent.

p	25%	$\mu-0.5\sigma$	$\mu-0.25\sigma$	μ	50%	$\mu+0.25\sigma$	$\mu+0.5\sigma$	75%
n	25%	$\mu-0.5\sigma$	$\mu-0.25\sigma$	μ	50%	$\mu+0.25\sigma$	$\mu+0.5\sigma$	75%
q	25%	$\mu-0.5\sigma$	$\mu-0.25\sigma$	μ	50%	$\mu+0.25\sigma$	$\mu+0.5\sigma$	75%
α_{ss}	1	1	1	**0.86**	0.95	0.65	0.51	0.38
c_{ss}	40.45	40.45	40.45	**36.03**	38.87	28.74	23.69	18.96

Values for mean prices used in the model are in bold. The model hits constraint, $0 \leq \alpha \leq 1$, when $\alpha_{ss} = 1$ and $c_{ss} = 40.45$.

Table 8: Sensitivity analysis: steady-state values for share of residues, α_{ss}, and soil carbon, c_{ss}, when changing one or two of prices (increasing or decreasing by 25% of the standard deviation, σ).

p	μ	μ	$\mu - 0.25\sigma$	μ	$\mu - 0.25\sigma$	$\mu - 0.25\sigma$
n	μ	$\mu - 0.25\sigma$	μ	$\mu - 0.25\sigma$	μ	$\mu - 0.25\sigma$
q	$\mu - 0.25\sigma$	μ	μ	$\mu - 0.25\sigma$	$\mu - 0.25\sigma$	μ
α_{ss}	1	0.74	0.77	1	1	0.66
c_{ss}	40.45	32.00	32.93	40.45	40.45	28.99
p	μ	μ	$\mu + 0.25\sigma$	μ	$\mu + 0.25\sigma$	$\mu + 0.25\sigma$
n	μ	$\mu + 0.25\sigma$	μ	$\mu + 0.25\sigma$	μ	$\mu + 0.25\sigma$
q	$\mu + 0.25\sigma$	μ	μ	$\mu + 0.25\sigma$	$\mu + 0.25\sigma$	μ
α_{ss}	0.49	0.995	0.96	0.58	0.56	1
c_{ss}	23.17	40.29	39.12	26.21	25.61	40.45

The model hits constraint, $0 \leq \alpha \leq 1$, when $\alpha_{ss} = 1$ and $c_{ss} = 40.45$.

A Appendix

A.1 Soil carbon stock value

The chronosequence data set contains 1,450 observations across 28 farms, eight years (2005, 2006, 2007, 2008, 2009, 2010, 2011, 2012) and four treatments, each with and without nitrogen fertilizer (control with and without N, *Tithonia diversifolia* leaves with and without N, wood charcoal with and without N, and sawdust with and without N). In addition, for 2011 and 2012 we have varied rates of N application on control plots – 0, 0.08, 0.12, 0.16, 0.20 and 0.24 N Mg/ha. Since the soil analysis of all observations was cost- and time-prohibitive, we used the following sampling design to select a sub-sample for detailed soil chemistry analysis. Three farms at each conversion group – old, medium-age, young – were chosen for their close geographic proximity.[23] Prior to analysis we mixed soil of the three farms at each conversion group for each year and treatment to create representative soil samples at each conversion group for the total number of 177 soil samples.

The soil samples were analyzed at the Cornell Nutrient Analysis Laboratory (CNAL) in May 2014 for total nitrogen and carbon as percentage by dry weight. To calculate soil carbon stocks on an equal mass basis, we follow the procedure described in Betemariam et al. (2011). Soil carbon stock is calculated by multiplying the carbon concentration of an oven-dry weight with bulk density and soil depth:

$$SC = \frac{C}{100} \times \rho \times D \times (1 - frag) \times 100, \tag{19}$$

where SC is soil carbon stock (Mg C/ha), C is soil carbon concentration of soil fines determined in the laboratory (%), ρ is soil bulk density (g/cm^3), D is depth of the sampled soil layer (cm), $frag$ is % mass fraction of coarse fragments/100, and 100 is used to convert the unit to Mg C/ha. We assume $frag$ to equal 5%. Soil bulk density was measured for all chronosequence farms in 2009 as described in Guerena (2014). We use the average (across three farms at each conversion group) bulk density for each treatment with and without fertilizer. Table A1 shows the average bulk densities used.[24]

Following Kinyangi (2008), we establish long-term carbon degradation equilibria of tropical soil (of 10 cm depth) as a function of time under long-term cropping. The soil analysis data are fitted using a three parameter single exponential decay model, $y = y_0 + ae^{-bx}$, where y_0 is the final soil carbon equilibrium level, a is C degradation (Mg/ha), b is rate of loss, and x is years since conversion from forest. We first fit the model using the full data set (177 soil samples), then fit the model for each of the treatment-fertilizer group separately.[25] The results are shown in Table A2

[23] Three farms in Kechire converted in 1950, three farms in Kereri/Bonjoge converted in 1986, and three farms in Sik Sik converted in 2000.

[24] Using average conversion group bulk density or average full sample bulk density does not significantly alter the results.

[25] We also tried fitting OLS to the data, with soil carbon stock as a dependent variable and indicator variables for treatment, year and farm age as independent variables, both for the full sample and for the three farm age groups separately. The exponential model provided, however, a better fit and out-of-sample predictions.

Table A1: Soil bulk density (g/cm^3).

Treatment	(1) Old farm	(2) Medium-age farm	(3) Young farm
Control=1, Fertilizer=0	1.2246	1.2187	1.2246
Control=1, Fertilizer=1	1.1300	1.2246	1.1297
Charcoal=1, Fertilizer=0	1.2246	1.1918	1.2246
Charcoal=1, Fertilizer=1	1.1244	1.1616	1.0903
Sawdust=1, Fertilizer=0	1.2246	1.1837	1.2246
Sawdust=1, Fertilizer=1	1.1153	1.1943	1.0279
Tithonia=1, Fertilizer=0	1.2246	1.1474	1.2246
Tithonia=1, Fertilizer=1	1.1433	1.1806	1.0495
Average by age	1.1491	1.1878	1.0980

Bulk density measured in 2009. **Full sample bulk density, 1.1450,** is the average of 72 observations. Average by age bulk density is the average of 24 observations in each age group with 8 treatments and 3 farms.

and Figures A1, A2, A3, A4, and A5.

Table A2: Exponential fit on full sample vs. for each treatment-fertilizer group.

Treatment	(1) y_0	(2) a	(3) b	(4) N	(5) R^2
Full sample	35.60	117.00	0.194	177	0.69
Control=1, Fertilizer=0	33.26	85.21	0.149	21	0.78
Control=1, Fertilizer=1	31.79	76.69	0.128	21	0.79
Charcoal=1, Fertilizer=0	40.20	262.93	0.307	21	0.75
Charcoal=1, Fertilizer=1	35.58	117.56	0.149	24	0.83
Sawdust=1, Fertilizer=0	36.08	230.11	0.275	21	0.82
Sawdust=1, Fertilizer=1	34.34	69.52	0.136	24	0.78
Tithonia=1, Fertilizer=0	37.61	106.26	0.178	21	0.64
Tithonia=1, Fertilizer=1	35.75	64.65	0.177	24	0.59

Based on $y = y_0 + a \times e^{-bx}$, where y is soil carbon stock (Mg/ha), y_0 is the equilibrium soil carbon level, a is carbon degradation, b is rate of loss, and x is years since conversion from forest.

Using parameters of the exponential decay model summarized in Table A2, we predict soil carbon stocks for the entire sample of 1,450 observations. The density of predicted soil carbon stocks (Mg/ha) is shown in Figure A6, with the correlation coefficient of the two predictions being 0.93. Given a slightly better fit of the model for each treatment-fertilizer group, we use the predictions shown on the right-hand side panel of Figure A6 in our estimation of the production function.

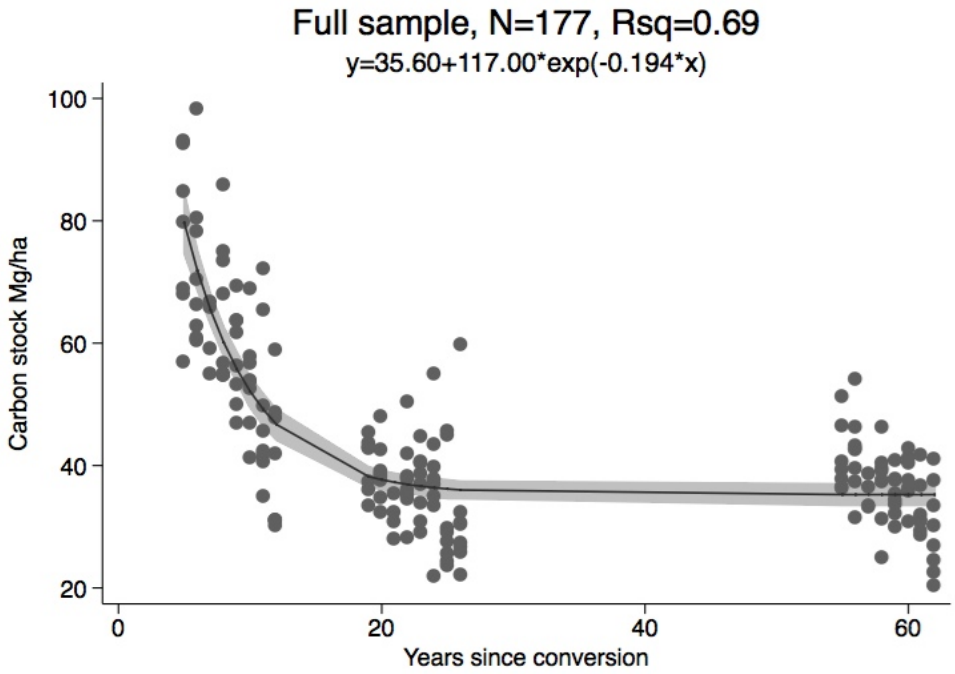

Figure A1: Exponential fit for full sample.

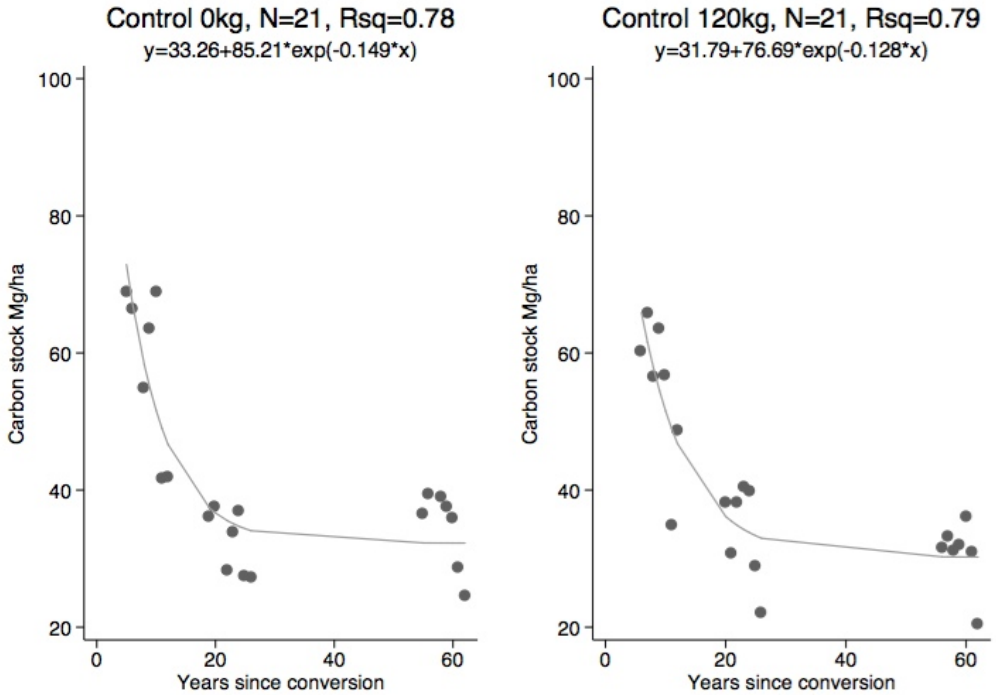

Figure A2: Exponential fit for control with and without N fertilizer.

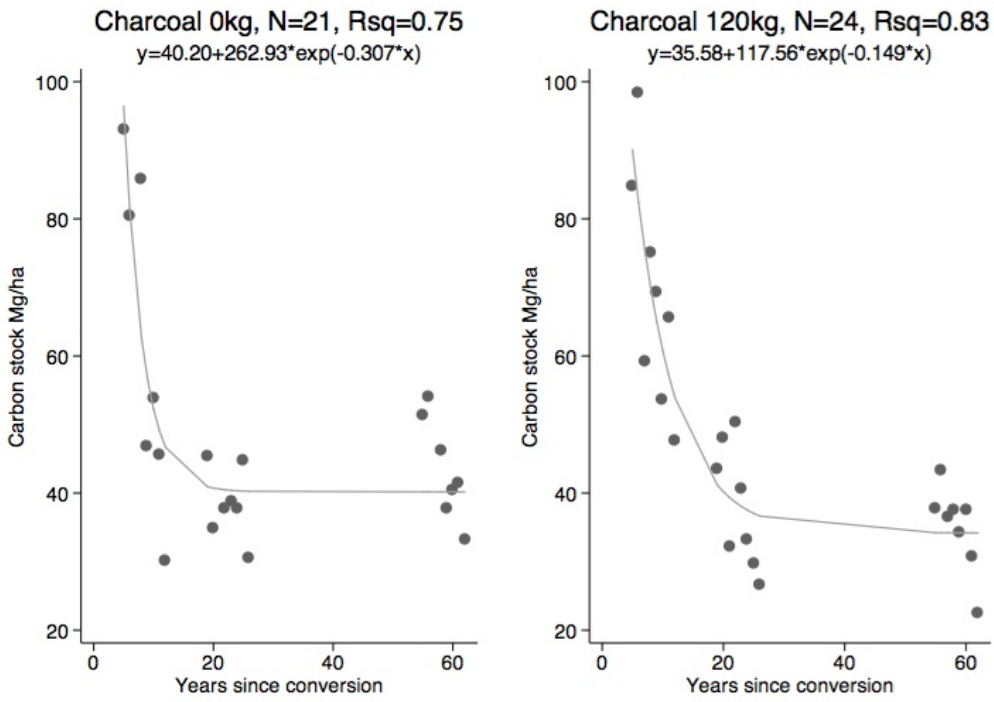

Figure A3: Exponential fit for charcoal with and without N fertilizer.

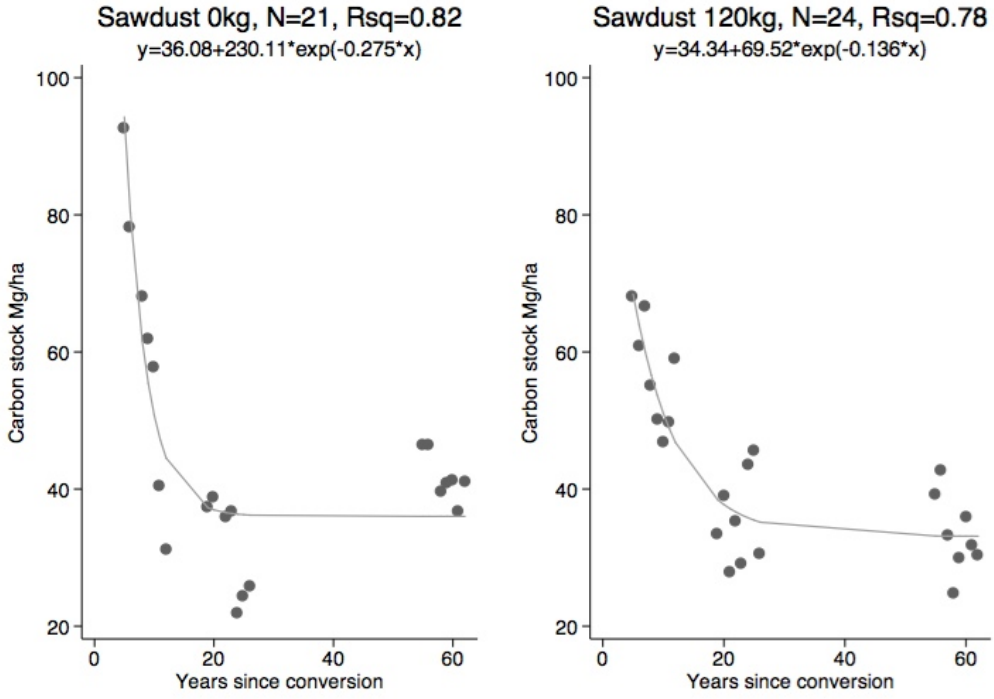

Figure A4: Exponential fit for sawdust with and without N fertilizer.

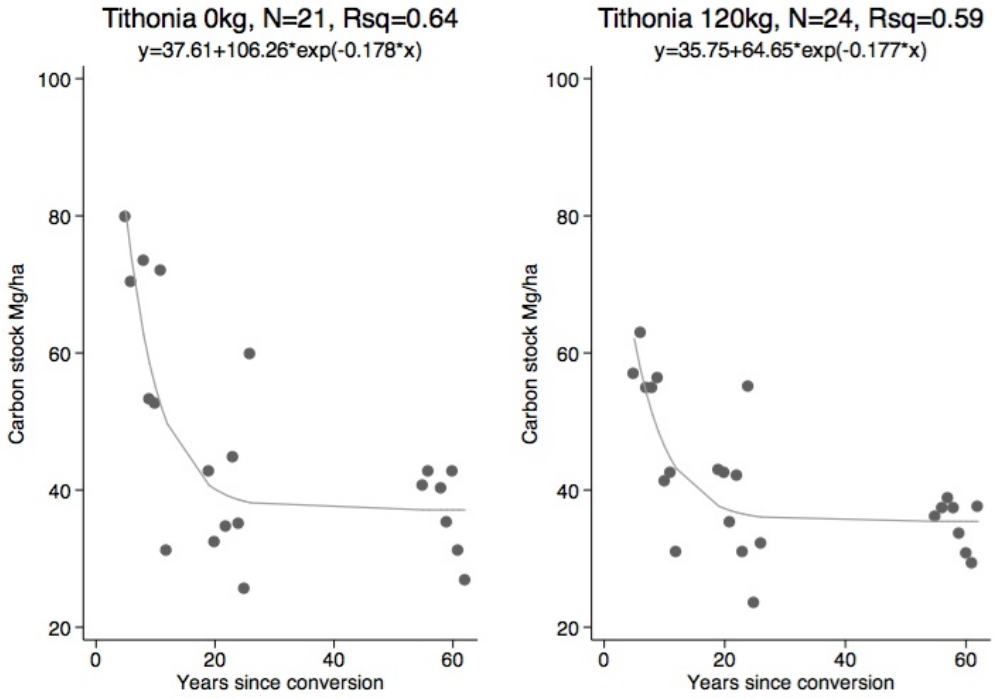

Figure A5: Exponential fit for *Tithonia* with and without N fertilizer.

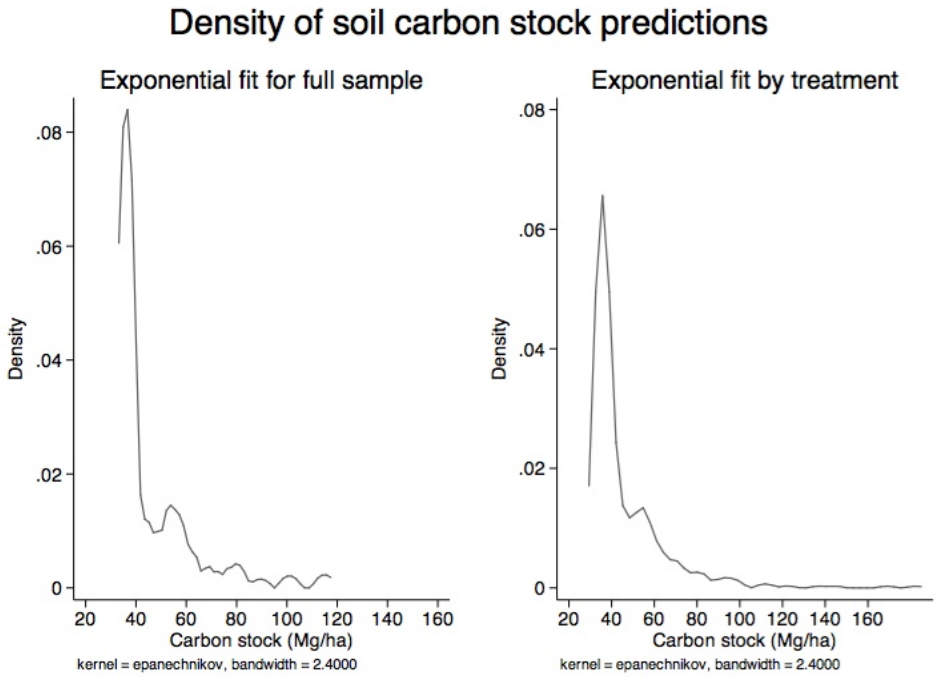

Figure A6: Density of predicted soil carbon stocks (Mg/ha).

A.2 Calibrating soil carbon equation

The Rothamsted Carbon Model (ROTHC-26.3) is one of the most widely used models to study the turnover of soil organic carbon. The model estimates the carbon dynamics for decades or centuries under different management and requires few inputs that are easily available for the chronosequence farms: the carbon inputs into the soil, as well as soil clay content and monthly weather conditions (rainfall, temperature, and open pan evaporation) (Coleman and Jenkinson 1999). The data requirements, their values and sources used for calibration of the Rothamsted carbon model are described in Table A3.[26]

Table A3: Data requirements for the ROTHC-26.3 model.

Variable	Data	Source
Average monthly mean air temperature (°C)*	18.2 (J), 18.2 (F), 18.5 (M), 19.6 (A), 19.7 (M), 19.5 (J), 20 (J), 20.2 (A), 20.3 (S), 20.2 (O), 18.8 (N), 18.6 (D)	New_locClim, FAO (2005)
Monthly precipitation (mm)*	129 (J), 201 (F), 170 (M), 150 (A), 97 (M), 80 (J), 58 (J), 117 (A), 127 (S), 243 (O), 207 (N), 150 (D)	New_locClim, FAO (2005)
Monthly evaporation (mm)*	132 (J), 144 (F), 146.13 (M), 158 (A), 145.07 (M), 160.27 (J), 163.73 (J), 154.80 (A), 172.53 (S), 144.13 (O), 138.27 (N), 130.27 (D)	New_locClim, FAO (2005)
Soil depth (cm)	10	
Clay content of the soil (%)	47	Kimetu et al. (2009)
DPM/RPM ratio for maize**	1.44	Coleman and Jenkinson (1999)
Soil cover	Fallow only in July and August	
Monthly input of plant residues	In February and June	
Amount of inert organic matter	Unknown, obtained as described in the modeling section	

*The weather data are displaced by six months so that the soil is at field capacity at the start of the model run, as suggested by Coleman and Jenkinson (1999). **Ratio of decomposable plant material to resistant plant material. Value suggested by Coleman and Jenkinson (1999).

The weather data come from New_LocClim: Local Climate Estimator, an FAO software program and database, which provides estimates of average climatic conditions at locations for which no observations are available (FAO 2005). The monthly long-term mean temperature, precipitation and evaporation are interpolated for the chronosequence farms – 00° 09' 34"N, 34° 57' 37"E, 1,780 m altitude (Kinyangi 2008).

ROTHC-26.3 is first run to estimate the equilibrium values for tropical forest with the climate data and soil clay content described in Table A3, and assuming annual inputs to the tropical forest soil equal to the mean net primary production of carbon for broadleaf evergreen forest specified in Potter (1999) – 1,075.4 g/m^2/year which translates to 0.896 Mg/ha/month. The initial level of measured total soil carbon is 65 Mg C/ha (Kinyangi 2008) and the DPM/RPM ratio, ratio of decomposable plant material to resistant plant material, is 0.25, the default for deciduous and tropical woodland (Coleman and Jenkinson 1999). Running the model in "inverse," when inputs are calculated from known changes in soil organic matter, we obtain the amount of inert organic

[26]The procedure used to calibrate the Rothamsted Carbon Model and estimate the equilibrium levels of soil carbon in the research area is developed by Dominic Woolf.

matter (IOM), 5.6899 Mg/ha, and monthly input of plant residues, 0.4996 Mg C/ha. Now running the model "forward," we obtain the starting values in Mg/ha for decomposable plant material (DPM) – 0.1410, resistant plant material (RPM) – 13.9892, microbial biomass (BIO) – 1.1866, humidified organic matter (HUM) – 43.9959, and inert organic matter (IOM) – 5.6899, to be used in the model simulations for different management scenarios.

To simulate the soil carbon equilibrium levels under different management scenarios, we run ROTHC-26.3 "forward" for different levels of plant carbon inputs (0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7, 7.5, 8 Mg C/ha) applied in February, the end of the main Long Rains season, and assuming the constant rate of plant carbon inputs, 1 Mg C/ha, in June to reflect the crop residues left on the field in the end of the Short Rains season. The resulting equilibrium levels of soil carbon (after 10,000 years) for different levels of plant carbon inputs are shown in Figure A7 that suggests a linear relationship: equilibrium soil carbon = 14.12431 + 8.247451 * plant carbon input.

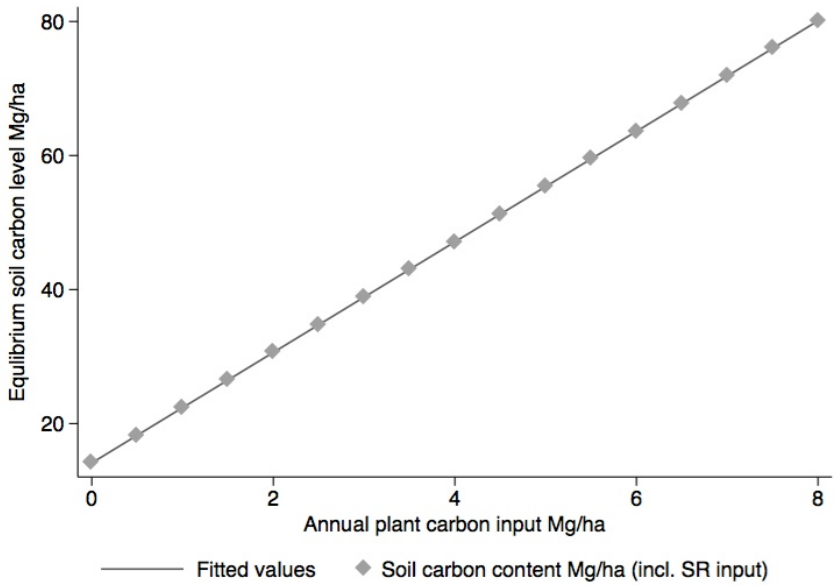

Figure A7: Equilibrium levels of soil carbon from ROTHC-26.3 as a function of plant carbon inputs.

Parameters A, β and M in soil carbon equation, $c_{t+1} - c_t = -Dc_t + A(\alpha_t M)^{\beta}$, are now chosen so that the steady-state carbon level in our model, c_{ss}, and the equilibrium carbon level calculated using ROTHC-26.3, c_{RothC}, are the same. The annual level of plant carbon input to determine c_{RothC} is determined using the steady-state maize yield, y_{ss}, and the steady-state share of crop residues, α_{ss}, assuming the average maize residues dry weight to be 1.9 of grain dry weight (Guerena 2014) and the carbon content of maize residues to be 40 percent (Latshaw and Miller 1924). The soil carbon analysis of the chronosequence data shows the same short-term equilibrium levels of soil carbon, as shown in Figure A1.

A.3 Prices

The following figures show the empirical distributions of prices as reported in the household survey or estimated in Berazneva et al. (2014), using the same household survey data. Table A4 shows the estimation of the hedonic price function used to approximate land rental values in the research area.

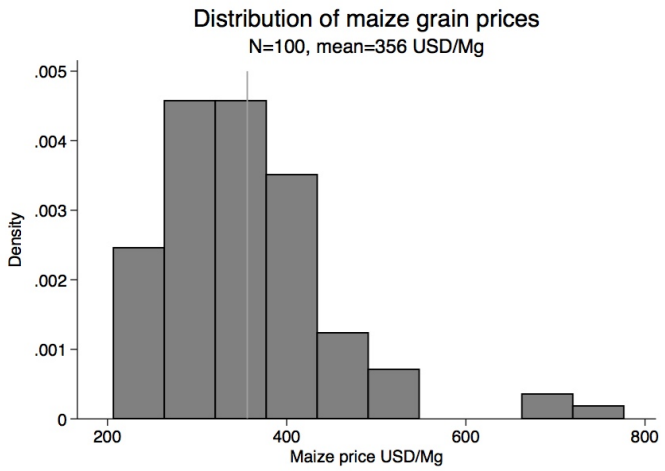

Figure A8: Maize grain price in the household survey (N=100, households that sold maize). The average market price in 2011-2012 is 410 $/Mg.

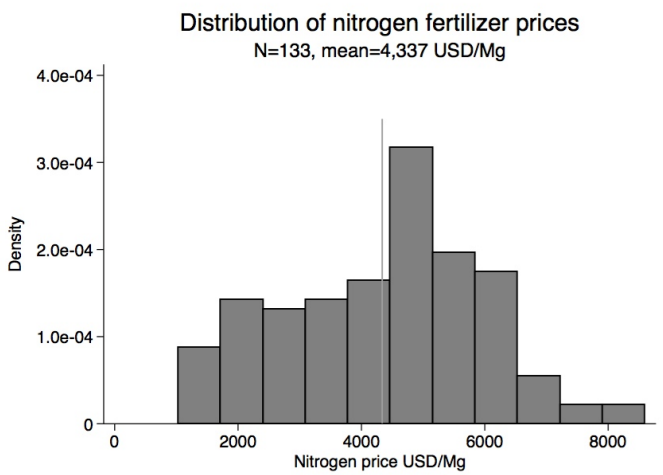

Figure A9: Nitrogen composite price in the household survey (N=133, households that bought DAP, urea and/or CAN). 2,070 $/Mg is the price of nitrogen in urea, 4,390 $/Mg is the composite price of nitrogen from the market surveys. n = price of DAP/0.18 $\times$ 0.69 + price of urea/0.46 $\times$ 0.16 + price of CAN/0.26 $\times$ 0.15.

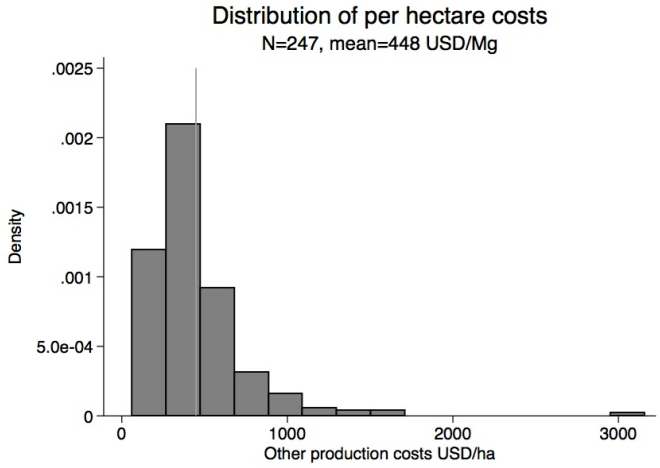

Figure A10: Per-hectare maize production costs of preparing the land, planting, and harvesting maize, including household and paid labor and land rental value (N=247).

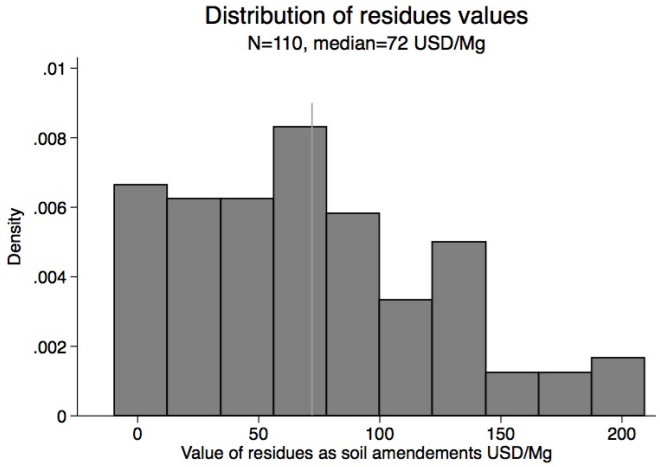

Figure A11: Value of maize residues left on the fields for soil fertility management as estimated in Berazneva et al. (2014) (N=110).

Table A4: Hedonic regression of land rental value.

Rent (KES/acre)	(1)	(2)
Indicator for loam soil	-668.6	-350.2
	(473.2)	(292.0)
Indicator for clay soil	-665.7	-781.6*
	(573.7)	(411.1)
Indicator for murram soil	-1,446***	-958.8***
	(512.9)	(329.7)
Indicator for moderate or fertile soil	912.4**	1,017***
	(388.4)	(289.2)
Organic soil carbon (% by weight)		52.47
		(164.7)
Plot GIS altitude (m)		2.168***
		(0.585)
Constant	2,113***	-1,435
	(512.9)	(1,011)
Observations	53	45
R-squared	0.170	0.481
Mean predicted value (st.dev.)	2,283.17 (502.18)	2,409.20 (846.54)
Median predicted value	2,356.39	2,363.18

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1. Indicator for sandy soil omitted.